# Year 12 Maths Advanced

Lesson 8
AP, GP and
Applications



## 1. Arithmetic and Geometric Progression

#### □ Overview

- Arithmetic and geometric progression (also known as APGP) can be found in practically every HSC exam. They often have their own questions where you must answer questions about a certain series, but are also heavily used in financial mathematics applications.
- It is vital that you are familiar with using the many formulas used in APGP and to be able to classify a given series as either arithmetic or geometric.

## □ Arithmetic Progression

- A sequence is arithmetic if  $T_n T_{n-1} = d$  where d is the common difference.
- The nth term of an arithmetic series is  $T_n = a + (n-1)d$  where a is the first term and d is the common difference
- The sum of the first n terms of an arithmetic series is  $S_n = \frac{n}{2}(a + T_n)$  if the last tmer of the series is given, otherwise by substitution,  $S_n = \frac{n}{2}(2a + (n-1)d)$

## ☐ Geometric Progression

- A series is geometric if  $\frac{T_n}{T_{n-1}} = r$  where r is the common ratio.
  - When testing if a series is geometric, prove:  $\frac{T_3}{T_2} = \frac{T_2}{T_1}$
- The nth term of a geometric series is  $T_n = ar^{n-1}$  where a is the first term and r is the common ratio
- $\blacksquare$  The sum of the first n terms of a geometric series is:

$$-S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$$
, where  $r \neq 1$ 

- We typically choose the formula based on whether r is greater than or smaller than 1.
- A limiting sum exists if |r| < 1. The limiting sum is:

$$S_{\infty} = \frac{1}{1 - r}$$

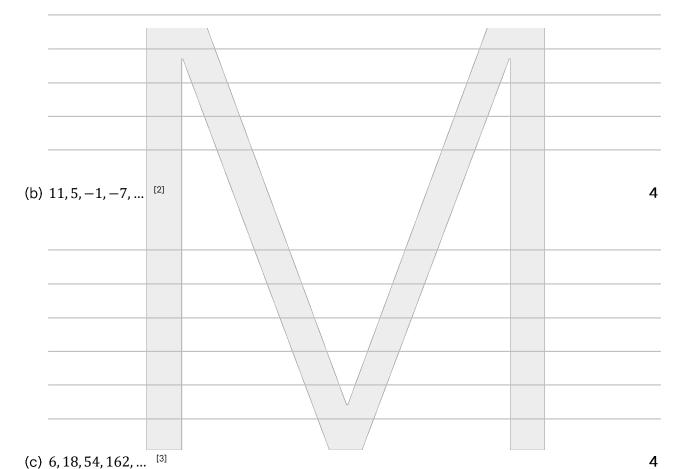
#### Concept Check 1.1

For the following series:

- (i) Identify if they are arithmetic or geometric.
- (ii) Find the value of d or r accordingly.
- (iii) Write the value of the 20th term.
- (iv) Find the sum of the first 15 terms.

(a) 
$$-7, -2, 3, 8, \dots$$
 [1]

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Our students come first 267

(٦)	3 3	3 3	[4]
(d)	$-\frac{1}{2},\frac{1}{4},\frac{1}{4}$	$-\frac{1}{8}, \frac{1}{16}, \dots$	

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(e) -	-2a, b,	2a +	2 <i>b</i>	,4a+	3b,	····	[5

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(f)	$3a^2, a, \frac{1}{3}, \frac{1}{9a}, \dots$	[6]
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### Concept Check 1.2

(1)	Find the values of $a$ and $d$ for this series. [7]	
(ii)	What is the sum of the first 25 terms? [8]	
(iii)	What is the sum of the next 20 terms AFTER the 10 <sup>th</sup> term? <sup>[9]</sup>	

(b) The sum,  $S_n$ , of the first n terms of a sequence  $u_1, u_2, u_3, ...$  is given by:

$$S_n = kn^2 + (5 - k)n$$

Where k is an unknown constant.

(i) Find  $u_n$  in terms of n and k. [11]

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(ii) Hence show that the sequence is an arithmetic progression. State the values of a and k for this series. [12]

The rth term,  $v_r$ , of another sequence  $v_1, v_2, v_3, ...$  is given by  $v_r = e^{u_r}$ 

(iii) Show that this sequence is geometric. State the value of the common ratio. [13]

(iv) Hence determine the values of k for which the sum to infinity exists. [14]  ${f 2}$