Year 12 Maths Advanced

Lesson 8 AP, GP and Applications

MATRIX EDUCATION

1. Arithmetic and Geometric Progression

☐ Overview

- Arithmetic and geometric progression (also known as APGP) can be found in practically every HSC exam. They often have their own questions where you must answer questions about a certain series, but are also heavily used in financial mathematics applications.
- It is vital that you are familiar with using the many formulas used in APGP and to be able to classify a given series as either arithmetic or geometric.

☐ Arithmetic Progression

- A sequence is arithmetic if $T_n - T_{n-1} = d$ where d is the common difference.
- The nth term of an arithmetic series is $T_n = a + (n-1)d$ where a is the first term and d is the common difference
- The sum of the first n terms of an arithmetic series is $S_n = \frac{n}{2}(a + T_n)$ if the last tmer of the series is given, otherwise by substitution, $S_n = \frac{n}{2}(2a + (n-1)d)$

☐ Geometric Progression

- A series is geometric if $\frac{T_n}{T_{n-1}} = r$ where r is the common ratio.
  - When testing if a series is geometric, prove: $\frac{T_3}{T_2} = \frac{T_2}{T_1}$
- The nth term of a geometric series is $T_n = ar^{n-1}$ where a is the first term and r is the common ratio
- The sum of the first n terms of a geometric series is:
  - $S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$, where $r \neq 1$
  - We typically choose the formula based on whether r is greater than or smaller than 1.
- A limiting sum exists if $|r| < 1$. The limiting sum is:

$$S_\infty = \frac{1}{1-r}$$

Concept Check 1.1

For the following series:

(i) Identify if they are arithmetic or geometric.

(ii) Find the value of d or r accordingly.

(iii) Write the value of the 20th term.

(iv) Find the sum of the first 15 terms.

(a) $-7, -2, 3, 8, \ldots$ [1] **4**

(b) $11, 5, -1, -7, \ldots$ [2] **4**

(c) $6, 18, 54, 162, \ldots$ [3] **4**

(d) $-\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, \ldots$ [4] **4**

(e) $-2a, b, 2a + 2b, 4a + 3b, \ldots$ [5] **4**

(f) $3a^2, a, \frac{1}{3}, \frac{1}{9a}, \ldots$ [6] **4**

Concept Check 1.2

(a) The 4th and 12th terms of an arithmetic series are 16 and 72 respectively.

(i) Find the values of a and d for this series. [7] **2**

(ii) What is the sum of the first 25 terms? [8] **2**

(iii) What is the sum of the next 20 terms AFTER the 10th term? [9] **2**

(iv) If this was instead, a geometric series, find the value r. [10] **2**

(b) The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by:

$$S_n = kn^2 + (5 - k)n$$

Where k is an unknown constant.

(i) Find u_n in terms of n and k. [11] **2**

(ii) Hence show that the sequence is an arithmetic progression. State the values of a and k for this series. [12] **2**

The rth term, v_r, of another sequence $v_1, v_2, v_3, \ldots$ is given by $v_r = e^{u_r}$

(iii) Show that this sequence is geometric. State the value of the common ratio. [13] **2**

(iv) Hence determine the values of k for which the sum to infinity exists. [14] **2**