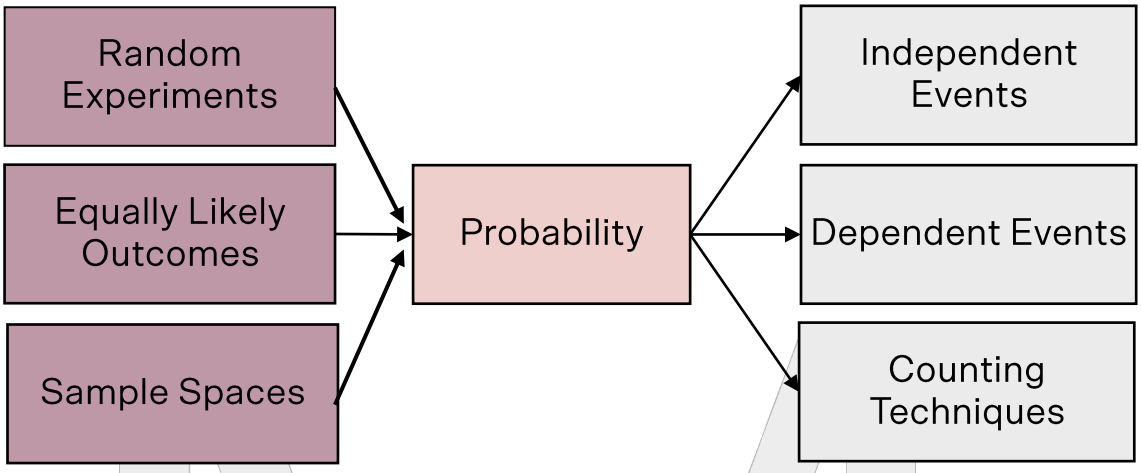
# Year 11
# Maths Advanced

# Lesson 3
# Probability 1

MATRIX
EDUCATION

# 1. Overview of Probability

## ☐ Overview of Probability




- In the next two lessons we will examine the fundamentals of probability theory and look at two broad question types:
  - Independent events (e.g. tossing dice)
  - Dependent events (e.g. choosing socks from a drawer)
- It is fair to say that many students struggle a little with this topic and often find it difficult to understand what they are being asked to do. A couple of general tips are:
  - Read the question a few times. Probability questions are often very descriptive and you need to wrap your brain around the problem.
  - A 2 Unit probability problem will almost always fall neatly into the category of Sample space, Tree diagram or Venn diagram question. Start by considering the type of problem you are facing.
  - If appropriate, use props such as tree diagrams, Venn diagrams and sample space pictures.
  - If it looks as though you are going to have to make an unusually large number of calculations consider the possibility of dealing instead with the opposite complementary problem. In particular $P(at\ least\ one) = 1 - P(none)$.
  - If you find yourself using the word “and”, multiply the probabilities. If it’s “or”, add the probabilities or in more complex situations use Venn diagrams.

# 2. Theory of Probability

## ☐ Sample Space and Events

- The set $S$ of all the possible outcomes of a given random experiment is called the **sample space.**
- A particular outcome, i.e. an element of S, is called a ***sample point or sample.***
- An **event** $A$ is a set of outcomes that is part of the sample space, i.e. Event $A$ is a subset of the sample space.

**Did you know?**

The sample space is **everything** that could have happened

## ☐ Probability of an Event

- In probability, the sample space, $S$, is all the possible outcomes in the random experiment and an event is the list of favourable outcomes.
- If the outcomes of an event are equally likely, then the probability of the event occurring is given by:

**Probability of an Event**

$$P(event) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{n(E)}{n(S)}$$

- For any event , $\boldsymbol{n(E) \leq n(S)}$, hence $\boldsymbol{0 \leq P(A) \leq 1}$
- When $\boldsymbol{P(A) = 0}$, then it is impossible for event $\boldsymbol{A}$ to occur in the random experiment.
- When $\boldsymbol{P(A) = 1}$ then event $\boldsymbol{A}$ is a certainty to occur in the random experiment, i.e. $\boldsymbol{n(E) = n(S)}$.

## Concept Check 2.1

(a) A letter is chosen at random from the letters of the word $EXPERIENCE$. What is the probability that the letter is:

(i) An $E$? [1] 1

(ii) A vowel? [2] 1

(b) Two hundred people were selected at random from an electoral roll and were asked "Are you in favour of capital punishment?" Their responses are recorded in the table below:

| | Men | Women |
|---|---|---|
| Yes | 35 | 20 |
| No | 40 | 65 |
| Undecided | 15 | 25 |

A person was selected at random from this group of 200. What is the probability that the selected person was:

(i) A man. [3] 1

(ii) A man against capital punishment. [4] 1

(iii) A woman in favour of capital punishment. [5] 1

(iv) Undecided or in favour of capital punishment. [6] 1

## Concept Check 2.2

(a) A bag contains black and white discs in the ratio 3 : 7. A disc is selected at random from the bag. What is the probability that the disc is white? [7] 1

(b) Computer chips produced by a particular manufacturing company were tested for quality control purposes. A sample of 250 chips was tested and 5 were found to be faulty. Repeating the same test on other samples of 250 chips produced the same result.

(i) Based on this result, what is the probability that a chip produced by this manufacturing company is not faulty? [8] 1

(ii) The company exports its chips in consignments of 10 000. How many faulty chips would you expect to find in each consignment? [9] 1

(c) Gordon has a pond full of goldfish in his backyard. He wanted to find out how many goldfish he had so he caught 60 of them, tagged them and released them back into the pond. The next day, he caught 100 goldfish and found 12 of them were tagged. He repeated this process several times during the day and found the same result.

(i) Estimate the probability of catching a tagged fish in Gordon's pond. [10] 1

(ii) Estimate the fraction of tagged fish in Gordon's pond. [11] 1

(iii) Estimate the number of fish in Gordon's pond. [12] 1

**Did you know?**

This is called the capture-recapture technique to estimate the size of a population.