Year 12
Maths Extension 1

Lesson 6
Revision – Permutations and Combinations, PHP & The Binomial Expansion

MATRIX
EDUCATION

1. Permutations

☐ Ordered Arrangements with Repetition

- The topic of permutations develops efficient techniques to determine the number of different ways certain events can happen.
  - The events will occur in succession and the order in which they occur **matters**.
- In counting ordered selections with repetition allowed, the number of objects to choose from does **not** change as each event is completed.
- Using the multiplication principle, we can imagine each of the n objects being able to be placed into each of the k boxes, and so the number of selections is:

Ordered arrangements of n objects into k boxes with repetition is n^k

☐ Ordered Arrangements without Repetition

- In counting ordered selections without repetition, the number of objects to choose from decreases by 1 as each event is completed.
- This means that the number of arrangements of n objects into k boxes will be $n \times (n-1) \times (n-2) \times \dots . \times (n-k+1)$. This is sometimes written as

Ordered arrangements of n objects into k boxes without repetition

$$n(n-1)(n-2)\dots(n-k+1) = {}^nP_k$$

- ${}^n\boldsymbol{P}_k$ stands for **"n permute k"**.

Concept Check 1.1

(a) A group of 6 friends including Selina and Don are sitting on a couch watching tv as demonstrated below. **1**

How many ways can Selina be sitting anywhere to the left of Don? [1]

(b) A car number plate has three letters of the alphabet followed by three digits selected from the digits 1,2,...,9. How many different number plates are possible if repetition of the letters and digits is not allowed? [2] **2**

(c) Using the letters of the word $SHARPEN$, how many:

(i) Three-letter codes can be formed. [3] **1**

(ii) Four-letter codes can be formed. [4] **1**

(iii) Six-letter codes can be formed without repeating any letter? [5] **1**

☐ Notation for Permutation, ${}^nP_k$

- We now want to find a concise form for

$$ {}^nP_k = n \times (n-1) \times (n-2) \times \ldots \times (n-k+1) $$

- Remembering our factorial notation learnt in the Binomial Theory lessons:

$$ n! = n \times (n-1) \times (n-2) \times \ldots \times (n-k+1)(n-k)(n-k-1) \times \ldots \times 3 \times 2 \times 1 $$

and

$$ (n-k)! = (n-k)(n-k-1) \times \ldots \times 3 \times 2 \times 1 $$

- So, multiplying top and bottom of ${}^nP_r$ by $(n-r)!$, we see that:

$$ {}^nP_k = \frac{n \times (n-1) \times (n-2) \times \ldots \times (n-k+1) \times (n-k)!}{(n-k)!} $$

$$ = \frac{n!}{(n-k)!} $$

$$ \boldsymbol{{}^nP_k = \frac{n!}{(n-k)!}} $$

☐ Ordered Arrangements involving Restrictions

- Let us first assume there are n different element to be arranged in a line. The number of ordered selections (without replacement) of n elements from a set of n elements is $\boldsymbol{n \times (n-1) \times (n-2) \ldots \ldots 3 \times 2 \times 1 = n!}$
- This is called a PERMUTATION of n elements selected from a set of n different elements.

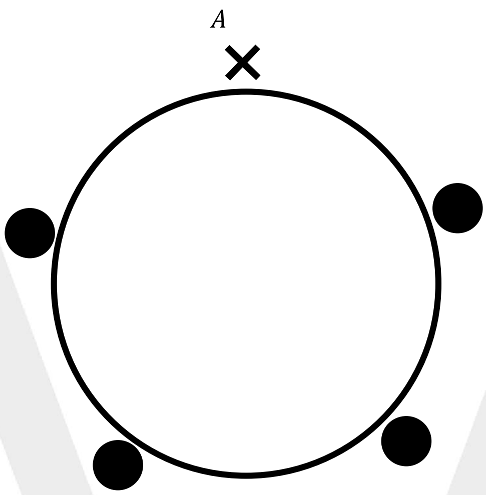
Number of Ordered Arrangements of n Objects Without Restriction

$$ = n! $$

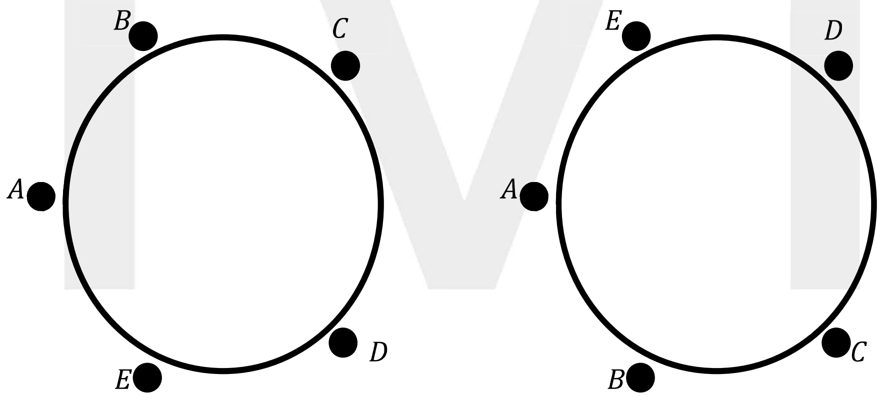
- Note that the most common type of permutation and combination questions involve a restriction on how we can arrange our set.
- When a problem presents to you any restrictions, always deal with any restrictions first, then arrange the remaining elements.

☐ Further Review of Permutations

- The arrangement of $\boldsymbol{n}$ elements where one element is before another element is $\frac{n!}{2}$
  - Eg: Arrangements of the set $\{A, B, C, D, E\}$ where A is before B is $\frac{5!}{2}$.
- The number of arrangements of n objects in a circle is $(n-1)!$



- The number of arrangements of n objects on a necklace is $\frac{(n-1)!}{2}$.
  - The clockwise and anti-clockwise arrangements of the beads **are the same arrangements.**



Concept Check 1.2

(a) In how many ways can 4 boys and 3 girls be arranged in a line if:

(i) There are no restrictions? [6] **1**

(ii) The boys and girls alternate? [7] **1**

(iii) Boys and girls are in separate groups? [8] **1**

(iv) Two particular boys X and Y want to remain together? [9] **1**

(b) The letters of the word $EXTENSION$ are arranged in a straight line. How many ways can this be done, if:

(i) There are no restrictions? [10] **1**

(ii) The word is to commence and end with the same letter? [11] **2**

(iii) The word had the two Es together? [12] **2**

(c) In George's cupboard there are three black and four white shirts that are identical in size and shape except for their colour. The shirts are hung on a rack in a line.

(i) George is very disorganised and hangs the shirts up in random order. How many different arrangements of the 7 shirts are possible? [13] **1**

(ii) How many different arrangements of just four of these shirts are possible? [14] **1**

Concept Check 1.3

(a) The 4 boys and 4 girls are seated around a circular table. How many different ways can the boys and girls be arranged if:

(i) There are no restrictions. [15] **1**

(ii) At least two of the girls are together. [16] **2**

(iii) All the boys are together. [17] **2**

(b) Four married couples are to be seated around a circular table for dinner.

(i) In how many different ways can the people be seated around the table? [18] **1**

(ii) If each married couple is to be seated together, in how many ways can this be done? [19] **2**

(c) In how many ways can 12 people be arranged around two circular tables of six? Assume that the two tables are distinguishable. [20] **2**

(d) How many ways are there of arranging 10 different marbles:

(i) In a line? [21] **1**

(ii) In a circle? [22] **1**

(iii) If a string is sewn through all of the marbles to create a necklace. How many different necklaces can be made? [23] **1**