Year 11 Maths Advanced

Lesson 8 Absolute Values

MATRIX EDUCATION

3. Graphs of Absolute Values

☐ Sketching Absolute Value Graphs

Absolute value graphs:

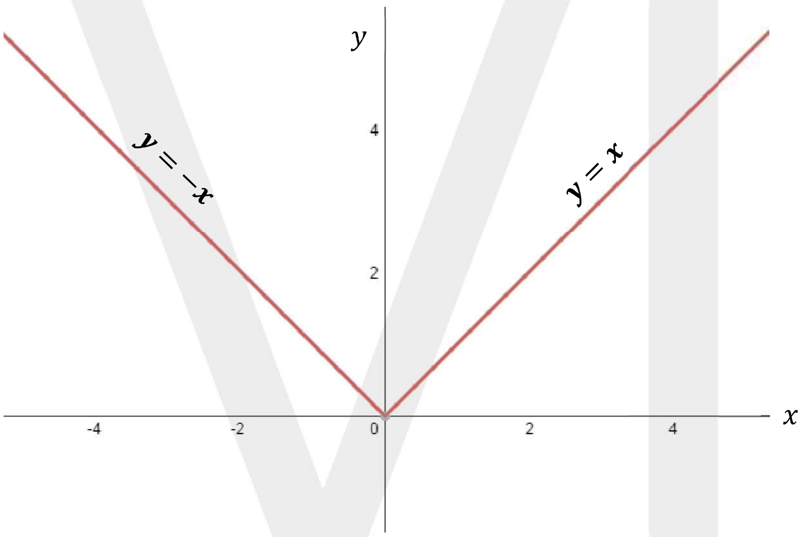
1) Simplify the expression involving absolute values for each restriction on x.

2) Sketch each "piece" of the graph in the restricted domains and label the branches.

- The standard absolute value curve $y = |x|$ looks like a "V" with a cusp at the origin.
- This absolute value can be simplified with restrictions:

$$|x| = \begin{cases} x; & x > 0 \\ 0; & x = 0 \\ -x; & x < 0 \end{cases}$$

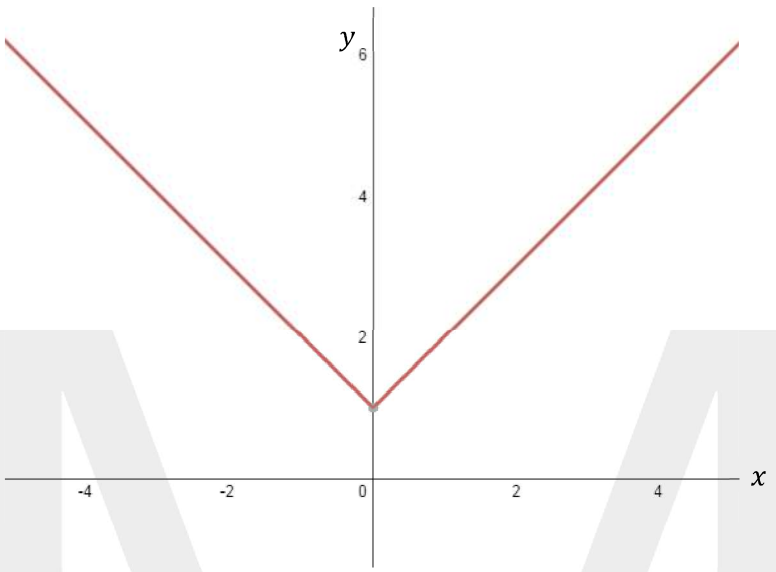
  – Therefore the two branches represent the lines $y = x$ and $y = -x$.



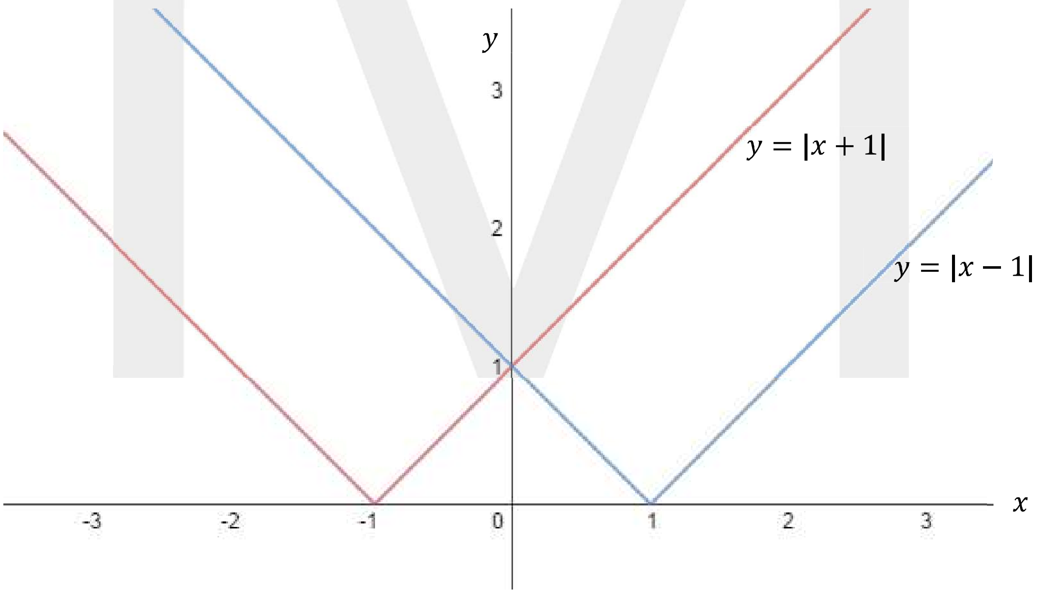
Concept Check 3.1

Label the branches of each of the following absolute value graphs

(a) $y = |x| + 1$



(b) $y = |x + 1|$ and $y = |x - 1|$

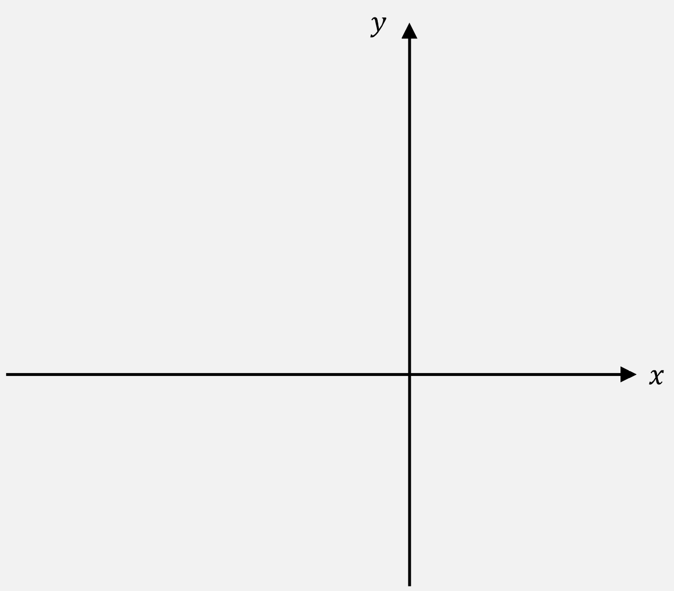


Example 1: Draw the graph of $y = |x + 2|$ [16]

Solution:

Step 1: **Simplify the expression for each restriction that is placed on x**

Step 2: **Draw the different graphs in each of the restricted domains and label the branches**



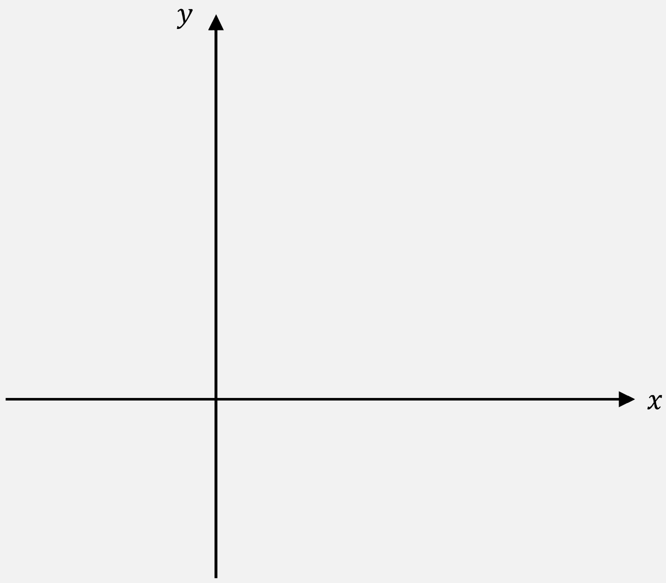
Note to students

If you know how to sketch the curve using knowledge of **graphical transformations** (to be studied in-depth next term), you can avoid investigating each restriction.

Example 2: Draw the graph of $x = |y - 3|$ [17]

Solution: Simplify the expression for each restriction that is placed on x

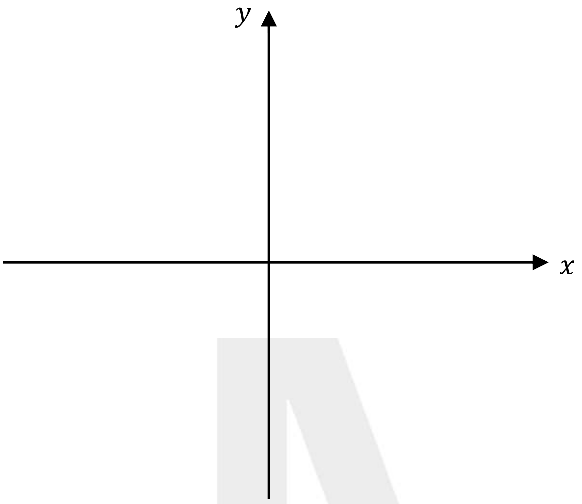
Draw the different graphs in each of the restricted domains and label the branches



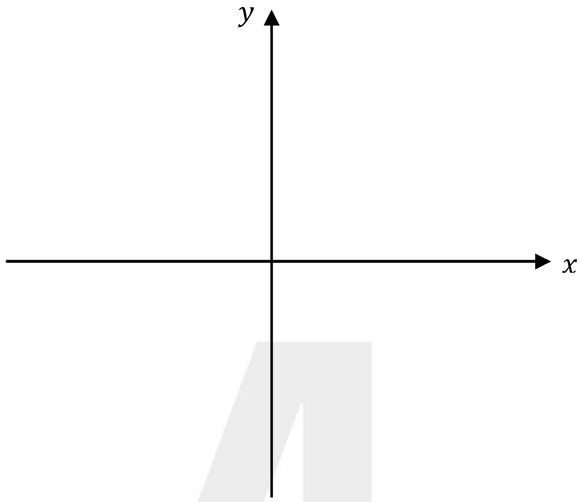
Concept Check 3.2

Sketch the following graphs on the coordinate plane, showing all intercepts.

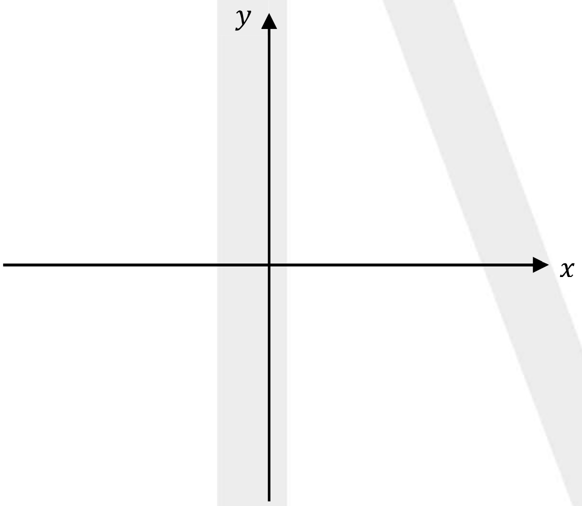
(a) $y = |x - 3|$ [18] **1**



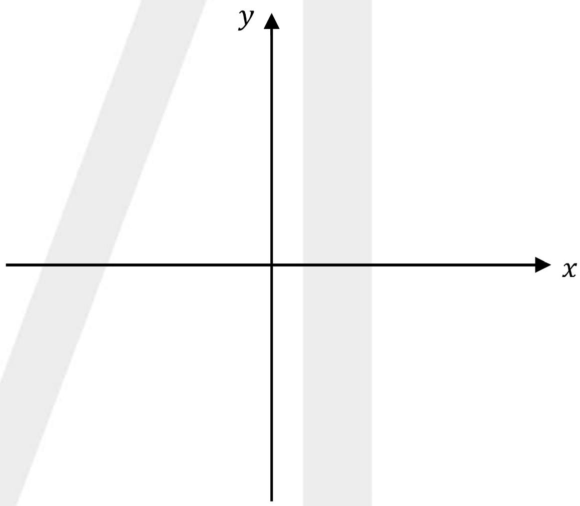
(b) $y = |3x + 2|$ [19] **1**



(c) $y = 2|x|$ [20] **1**



(d) $y = 2 - |x|$ [21] **1**



Discussion

Is $|1 - 2x|$ the same as $|2x - 1|$? [22]

4. Equations involving Absolute Values

☐ Algebraic Approach

- Solving equations involving absolute values makes heavy use of the fact:

If $|a| = b$ then $a = b$ or $-a = b$

- Simply solve the two equations and make sure you **test the solutions.**

Example 1: Solve $|2x - 3| = 5$ [23]

Solution:

Step 1: **Remove the absolute value signs by considering the two cases.**

Solve these equations.

$2x - 3 = 5$	$-(2x - 3) = 5$
______________________	______________________
______________________	______________________

Step 2: Test the solutions

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Example 2: Solve $|2x - 3| = 3x + 1$ [24]

Solution:

Step 1: **Remove the absolute value signs by considering the two cases.**

Solve these equations.

$2x - 3 = 3x + 1$	$-(2x - 3) = 3x + 1$
______________________	______________________
______________________	______________________
______________________	______________________

Step 2: Test the solutions

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Concept Check 4.1

Solve the following equations. Remember to test your solutions.

(a) $|3x - 1| = 11$ [25] **2**

(b) $|4 - 2x| = -6$ [26] **1**

Note to students

This one can be done without algebra! Can an absolute value ever produce a negative answer?

(c) $|5 - 2x| = 7$ [27] **3**

☐ Using Restrictions

Solving absolute value equations using restrictions:

1) Simplify the expression involving the absolute value for each restriction on x.

2) Solve the equations.

3) Check the solution satisfies the restriction placed on x.

Example 1: Solve $|3 - x| = 2x$ [28]

Solution: **Simplify the expression involving the absolute value for each restriction on x**

When $x \geq 3$; $|3 - x| = -(3 - x) = -3 + x$

When $x \leq 3$; $|3 - x| = 3 - x$

Solve the equations

When $x \geq 3$;

When $x \leq 3$;

Check the solutions satisfy the restrictions on x in each case

Example 2: Solve $\frac{|x-2|}{x^2-4} = \frac{3}{4}$ [29]

Solution: **Simplify the expression involving the absolute value for each restriction on x**

When $x > 2$;

When $x < 2$;

Solve the equations

When $x > 2$;

When $x \le 3$;

Check the solutions satisfy the restrictions on x in each case