Year 09
Maths

Lesson 7
Equations 4

MATRIX
EDUCATION

1. Inequalities

☐ Understanding inequalities

- An inequality is an algebraic statement containing one or more **inequality symbols**:

$\boldsymbol{a > b}$	a is greater than b
$\boldsymbol{a \geq b}$	a is greater than or equal to b
$\boldsymbol{a < b}$	a is less than b
$\boldsymbol{a \leq b}$	a is less than or equal to b

Discussion

Are $p \geq q$ and $q \leq p$ equivalent statements? [1]

_____

_____

- Inequalities are sometimes called **inequations** because they simply replace the = symbol with an inequality symbol.
- Inequalities are solved in the same way as equations (using inverse operations) except there will be many solutions. For example:

Equation	Inequality
$x + 5 = 7$	$x + 5 > 7$
$-5 \quad -5$	$-5 \quad -5$
$x = 2$	$x > 2$

  - The **equation** $x + 5 = 7$ has only **one solution**, $x = 2$. This is the only value of x that satisfies the equation (makes the equation true).
  - However, when we the **inequality** the same way, we get $x > 2$, a statement that says that any value of x greater than 2 is a solution. There are many numbers greater than 2, thus there are **many solutions**!

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- A solution to an inequality can be verified by substituting the value of the pronumeral into the inequality. If it holds true, then we say the solution **satisfies the inequality.**

Example:

Test whether $x = -4$ and $x = 18$ satisfy the inequality $x - 5 > 7$.

Solution:

Test $x = -4$: $LHS = x - 5$

$= -4 - 5$

$= -9$

$\not> 7$, therefore $x = -4$ does NOT satisfy the inequality.

Test $x = 18$: $LHS = x - 5$

$= 18 - 5$

$= 13$

> 7, therefore $x = 18$ DOES satisfy the inequality.

Concept Check 1.1

Check whether the given values satisfy the inequalities.

(a) $5x + 2 \leq 7$ Test $x = 3, -2, 1$ [2]

(b) $\frac{b+4}{-2} \leq 2b$ Test $b = 2, -3, -6$ [3]

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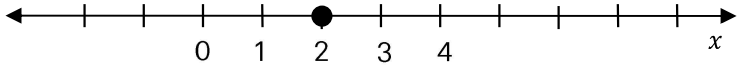
☐ Graphing inequalities on the number line

- The **solutions** of equations and inequalities can be **represented visually** on a number line. Consider the following examples:

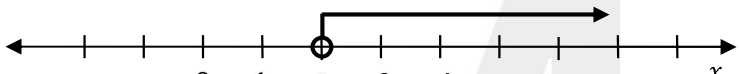
Equation

$$x + 5 = 7$$
$$-5 \quad -5$$
$$x = 2$$



Inequality

$$x + 5 > 7$$
$$-5 \quad -5$$
$$x > 2$$



Inequality

$$x + 5 \leq 7$$
$$-5 \quad -5$$
$$x \leq 2$$

- SUMMARY:
  - Arrows are drawn to the right to represent “greater than” and to the left to represent “less than”.
  - An **open circle** is used to represent strict inequalities $>$ or $<$. A **closed circle** is used to represent $\geq$ or $\leq$.

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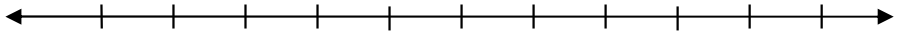
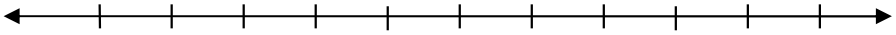
Concept Check 1.2

Show each of the following inequalities on the given number lines.

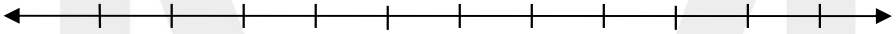
(a) $a \leq -8$



(b) $x > 5$



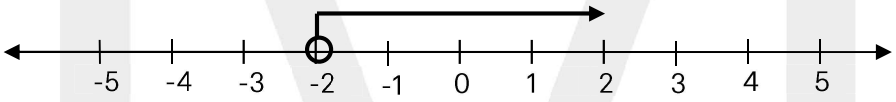
(c) $p < \frac{11}{3}$



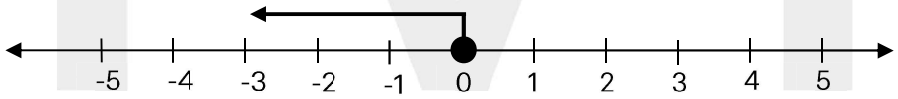
Concept Check 1.3

Write the inequality represented by each of the following graphs.

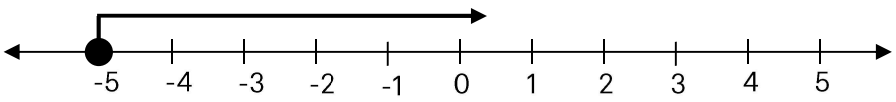
(a) [4]



(b) [5]



(c) [6]



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☐ Solving linear inequalities

- Inverse operations can be applied to inequalities in the same way they are applied to equations.

Discussion

Consider the inequality $6 > 4$. This is a true statement.

- Add 10 to both sides of the inequality. Is the new statement true?

  _____

- Subtract 11 from both sides of the inequality. Is the new statement true?

  _____

- Multiply both sides of the inequality by 8. Is the new statement true?

  _____

- Divide both sides of the inequality by 2. Is the new statement true?

  _____

- Multiply both sides of the inequality by -12. Is the new statement true?

  _____

- Divide both sides of the inequality by -2. Is the new statement true?

  _____

- What operations caused your inequality statement to become false? [7]

  _____

- What should happen to the inequality sign to keep the statement true? [8]

  _____

- When multiplying or dividing both sides of an inequality by a negative number, you must reverse the inequality sign.
  - The reverse of $<$ or $\leq$ is $>$ or $\geq$.
  - The reverse of $>$ or $\geq$ is $<$ or $\leq$.

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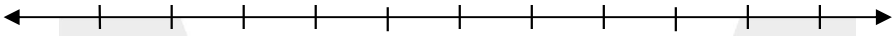
Concept Check 1.4

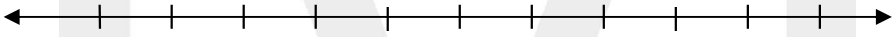
Solve the following inequalities and graph the solutions on the given number line.

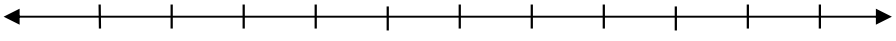
(a) $5x + 2 \geq 8$ [9]

(b) $\frac{6x}{5} - 1 > -7$ [10]



(c) $2a + 7 < -\frac{13}{3}$ [11]





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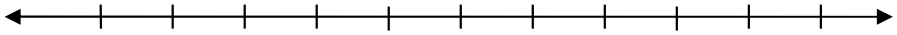
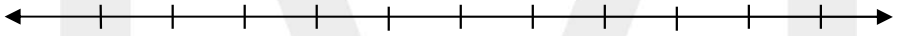
(d) $10 - 3t > 15 - t$ [12]



(e) $24 - 5q \leq 18 - 9q$ [13]



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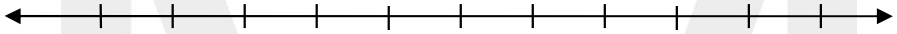
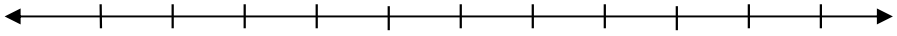
Concept Check 1.5

Solve the following inequalities and graph the solutions on the number line.

(a) $2(x-1)-4>12$ [14]



(b) $22x \geq 8(2x+3)+2$ [15]



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