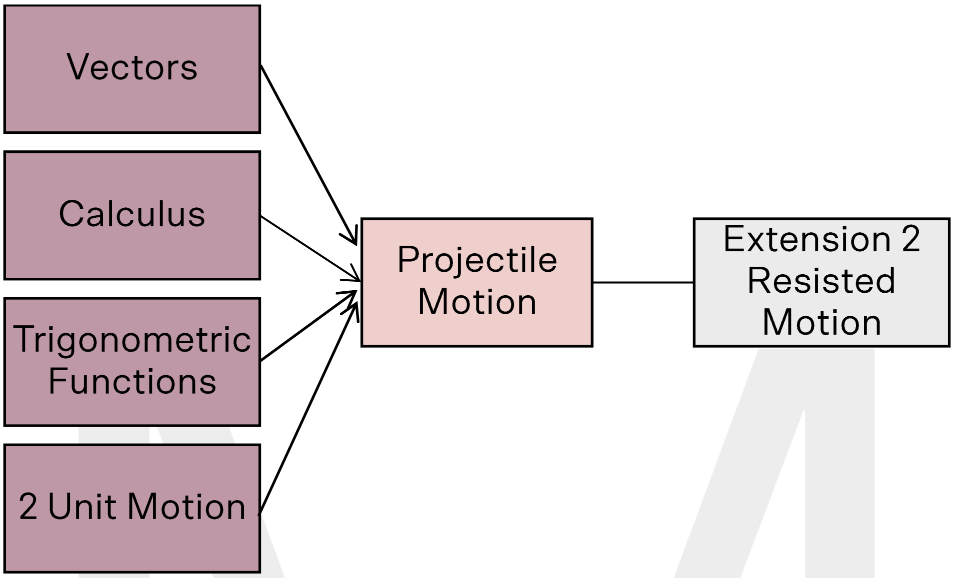
Year 12 Maths Extension 1

Lesson 1 Projectile Motion 1

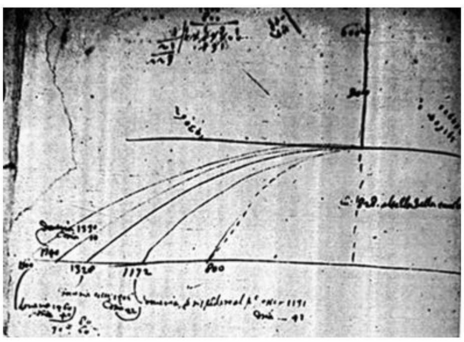
MATRIX EDUCATION

1. Overview of Projectile Motion

☐ Introduction to Projectile Motion



- Given the immediate and crucial military applications, it is not surprising that the analysis of projectile motion has an extensive and colourful history. Long before calculus, soldiers were trying to accurately predict the paths of their arrows and cannonballs. It was Galileo who first theorised that the path of a projectile was a parabola and this page from Galileo's personal notebook clearly displays the remarkable depth of his understanding. It was not until Sir Isaac Newton developed calculus however, that a full mathematical development of the theory of projectile motion was possible.
- Lessons 1 and 2 will use all of the tools of calculus together with our understanding of 2 unit motion and vectors to completely settle all of the issues surrounding projectile motion.
- By viewing projectile motion as a simple combination of horizontal and vertical motions we will be able to use integration and differentiation to calculate, for a projectile P, the:
  - time of flight.
  - horizontal range.
  - maximum height.
  - Cartesian equation of the path.
  - velocity and position of P at any time *t*.



2. Representing Two-Dimensional Motion

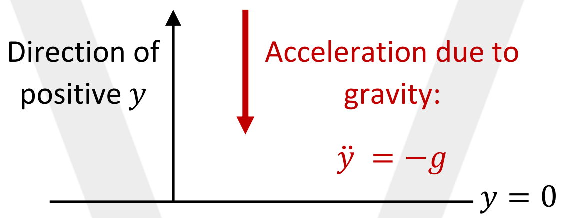
☐ Motions in Two-Dimensions

- We need to consider motion in both horizontal and vertical components.

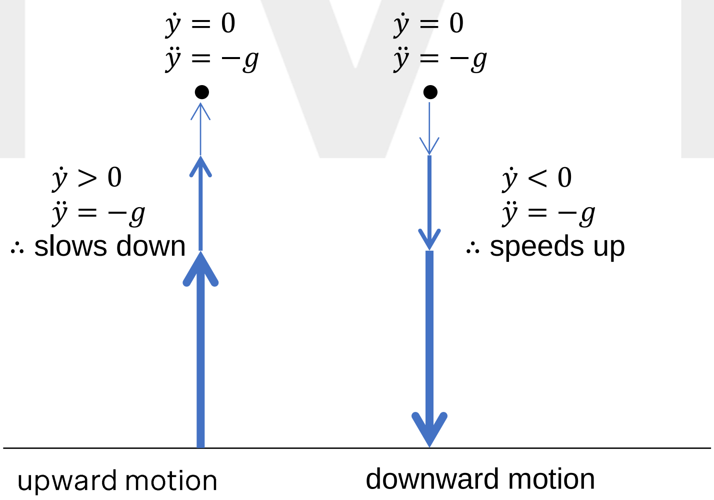
Horizontal displacement: x	Vertical displacement: y
Horizontal velocity: $\dot{x}$	Vertical velocity: $\dot{y}$
Horizontal acceleration: $\ddot{x}$	Vertical acceleration: $\ddot{y}$

☐ Vertical Projection

- When a body is projected in any direction, it is subject only to gravitational acceleration if we neglect the retardation associated with air resistance. The gravitational acceleration g acts downwards in the vertical direction, hence $a = -g$.



- We assign the upward direction as positive and the downward direction as negative. Neglecting the air resistance, the acceleration due to gravity ($g \approx 9.8$ m/s^2) acts downwards on the particle so $a < 0$.



Acceleration in the Vertical Direction is Constant:

$$\ddot{y} = -g$$

- For the upward motion the velocity is positive and the acceleration is negative. The velocity will decrease to zero then the particle reaches a maximum height, turns and returns to the ground.
- For the downward motion, the velocity and acceleration are negative so the magnitude of the velocity of the particle increases as it returns to the ground.

Note to students

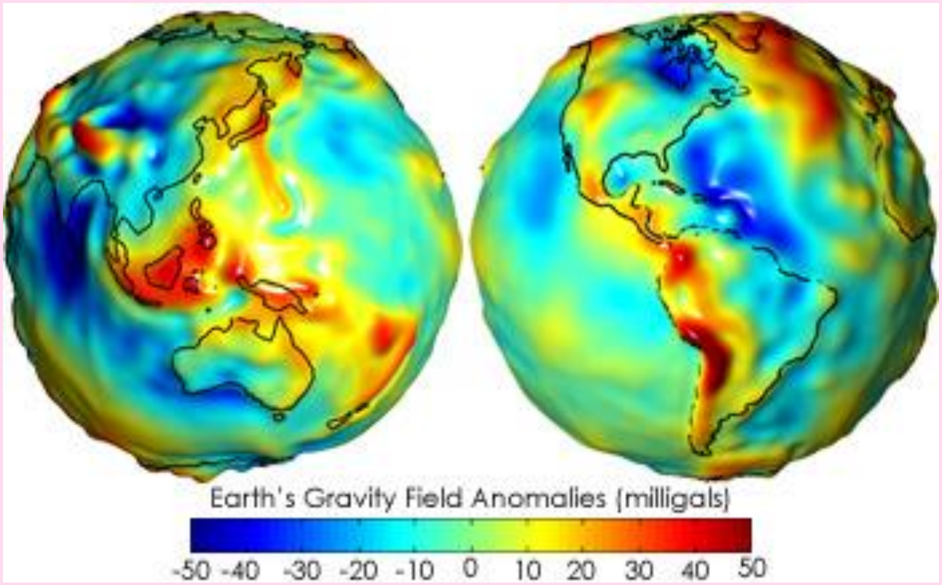
The direction of acceleration due to gravity remains unchanged and is independent of the direction of object's velocity:

- When direction of motion is upwards, acceleration acts downwards.
- When direction of motion is downwards, acceleration still acts downwards.

<u>Gravitational Acceleration ALWAYS acts downwards</u>

Did you know?

The acceleration due to gravity, g is not exactly uniform over the entire Earth; but fluctuates due to the Earth's shape and composition.



Credit: NASA GRACE mission

The acceleration due to gravity on Earth is always $\approx 9.81\ m/s^2$. This is often approximated as $g = 10\ m/s^2$.

Example

Maria threw a ball vertically upwards and caught it 1.6 seconds later. Assuming $g = 10\ ms^{-2}$ and the initial velocity is $8\ ms^{-1}$:

(a) Show that 2

$$\dot{y} = 8 - 10t, \qquad y = 8t - 5t^2$$

Step 1: State the equation of acceleration in the $\boldsymbol{y}$ direction

$\ddot{y} =$ ____________________

Step 2: Integrate the acceleration to find the velocity

$\dot{y} =$ ____________________

Step 3: Substitute in the given values to find the constant of integration.

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$\therefore \dot{y} =$......................................

Step 4: Integrate the velocity to find the displacement, and substitute in given values to find the constant of integration

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(b) Find the height the ball reached. [1] 2

- **This is also known as finding the "maximum height".**
- **The maximum height is when the ball must stop, change directions, and come falling back down to the ground**

$\therefore$ At the maximum height $\dot{y} =$ ____________________ [2]

- Use this substitution to find the time when it reaches the maximum height, and substitute the time into the $\boldsymbol{y}$ function.