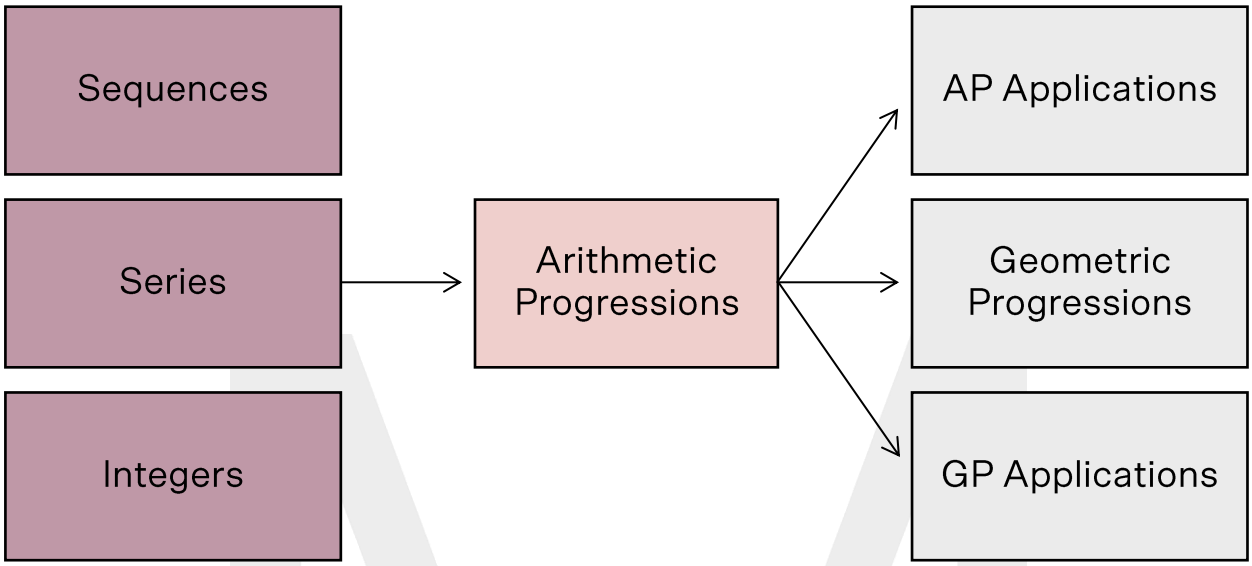
Year 12 Maths Advanced

Lesson 1 Arithmetic Series & Sequences

MATRIX
EDUCATION

1. Overview Of Arithmetic Series

☐ Introduction to Series and Sequences



- We begin Term 3 with the general theory of series and sequences. A sequence is simply a list of numbers, for example 1,4,7,10 and a series is what you get when you add these numbers up 1+4+7+10. The theory of series and sequences is vast and in the H.S.C. we restrict our attention to two very special types of sequences, Arithmetic Progressions (APs) and Geometric Progressions (GPs). More general sequences will be dealt with at university.
- For simpler sequence questions you will just be asked for:
  - The value of a particular term.
  - The value of a particular sum of terms.
  - A formula for the nth term.
  - Various features of the sequence.

- These sort of questions are easily mastered and implementation of a handful of standard formulae. The topic of APs and GPs does however get a little more difficult. You will sometimes be asked to use the theory to model complicated physical situations. In particular the theory of Geometric Progressions can be used to work out home loan repayments and superannuation lump sums. APs and GPs are part of the Advanced 2 Unit syllabus , however they often also appear in the Extension 1 paper as a harder 2 Unit question.
- In this first lesson we will look at APs only and try to get a feeling for the format and notation used in the topic.

2. Definition Of Sequence And Series

☐ Definition of a Sequence

- A sequence is a set of numbers that are arranged in some definite pattern or obeys a certain rule.

0, 1, 4, 9, 16, 25, 36, ...

- Each number in the sequence is called a term, T_n, where n is the numerical position of the term.
  - State the third term of the sequence.

  - Find the 8th term of the sequence.

- The rule that each term in the sequence obeys is called the generating formula of the sequence.
  - The generating formula of a sequence gives the formula for the n^{th} term of the sequence.
  - It is written T_n, where n stands for the number of terms in the sequence. The value of n is an integral value greater than or equal to 1.

Note to students

The T_n notation is similar to function notation $f(x)$. T_n is the n^{th} term of the sequence, e.g. T_5 is the 5th term of the sequence.

Note that $n \geq 1$ when examining counting numbers.

☐ Investigation

- Consider a sequence defined by $T_n = 2n - n^2$. Find:
  - The first term
  - The fifteenth term
  - T_{k+1}
  - T_r
  - the 5th term of the sequence

- A sequence that has a fixed number of terms is called a finite or restricted sequence.
  - Example of a finite sequence: 1, 3, 5, 7, 9.
- An infinite sequence is a sequence where the number of terms in the sequence increases without bounds.

  i.e. there are an infinite number of terms. It ends with "...".
  - Example of an infinite sequence: 2, 4, 6, 8, 10, 12, 14, 16, 18...

Concept Check 2.1

Write down the first four terms of the following sequences with n^{th} term defined by:

(a) $T_n = 2^{n-1}$ [1] 1

(b) $T_n = n + \frac{4}{n}$ [2] 1

(c) $T_n = (-1)^{n+1}n^2$ [3] 1

Note to students

$(-1)^n = 1$ where n is even

$(-1)^n = -1$ where n is odd

Concept Check 2.2

For each of the following sequences:

(a) Write down the next term

(b) Find the generating formula for the sequence by inspection.

(c) Use the generating formula to find the 50th term of the sequence.

Did you know?

We have two different ways of presenting a sequence.

1. We can just list the sequence: 1, 4, 7, 10,.. or
2. We can provide a generating formula

$$T_n = 3n - 2, \qquad n = 1,2,3, \ldots$$

Both are regularly used.

(i) 0, 1, 4, 9, 16, 25, 36, ... [4] 3

(ii) 1, 3, 5, 7, 9, 11, 13, ... [5] 3

Discussion question

What is the next term in this sequence: 3, -1, 4, 2, -7, 11, 1, ... [6]