YEAR 12 PHYSICS

ADVANCED MECHANICS

LESSON 2: PROJECTILE MOTION

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THE HSC EXPERTS

Syllabus Content

Projectile Motion

Inquiry question: How can models that are used to explain projectile motion be used to analyse and make predictions?

Students:

- analyse the motion of projectiles by resolving the motion into horizontal and vertical components, making the following assumptions:
  - a constant vertical acceleration due to gravity
  - zero air resistance

- apply the modelling of projectile motion to quantitatively derive the relationships between the following variables:
  - initial velocity
  - launch angle
  - maximum height
  - time of flight
  - final velocity
  - launch height
  - horizontal range of the projectile (ACSPH099)

- solve problems, create models and make quantitative predictions by applying the equations of motion relationships for uniformly accelerated and constant rectilinear motion

1. Projectile Motion

❒ What is projectile motion?

- An object moving **freely** under the **force of gravity** is called a **projectile.**
  - For an object to be a projectile, there must be no force other than the gravitational force acting on the object. Hence it moves "freely under the force of gravity."
  - The net force on the projectile is the gravitational force, i.e. its weight.
- All objects within the Earth's gravitational field will experience a gravitational force of attraction or force of gravity. As a result an object moving in the Earth's gravitational field will experience:
  - A vertically downwards force of 9.8 N/kg
  - A vertically downwards acceleration of 9.8 m/s^2

❒ Analysing projectile motion

- Projectile motion will be analysed using the equations of motion introduced in Year 11 – Module 1: Kinematics.
- However, projectile motion is **two dimensional motion**.
  - For the purpose of our study, we must break up the two–dimensional motion into **two one–dimensional motions**.
  - The horizontal motion (along x) and vertical motion (along y) are **independent** of each other (neither motion affects the other) so can be treated separately.
  - There is **no acceleration is the horizontal direction**, as gravity only acts vertically. This gives $a_x = 0$.
  - The equations used in projectile motion are:

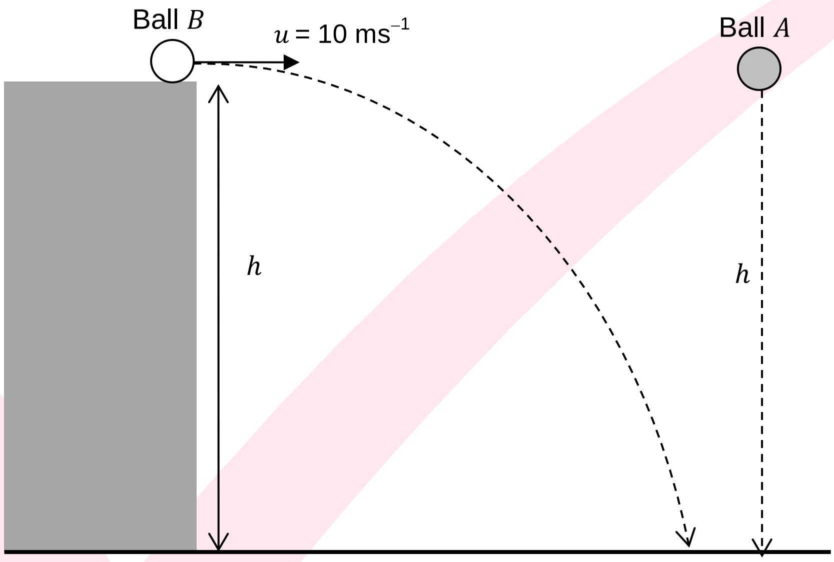
Horizontal Direction (x)	Vertical Direction (y)
$v_x = u_x$	$v_y^2 = u_y^2 + 2a_y\Delta y$
$\Delta x = u_x t$	$\Delta y = u_y t + ½ a_y t^2$
	$v_y = u_y + a_y t$

NOTE TO STUDENTS:
The equations of motion in the horizontal direction are the same as the ones in the vertical direction, but set with $a_x = 0$.

- We are only able to solve two dimensional projectile motion problems by separating the given data into horizontal and vertical components and working either horizontally or vertically, **never both at the same time**.

- The following **assumptions** are made:
  - The curvature of the Earth is ignored.
  - The **frictional resistance of the atmosphere** (air resistance) is assumed to be negligible and ignored.

- Compare the motion of the balls A and B.[1]

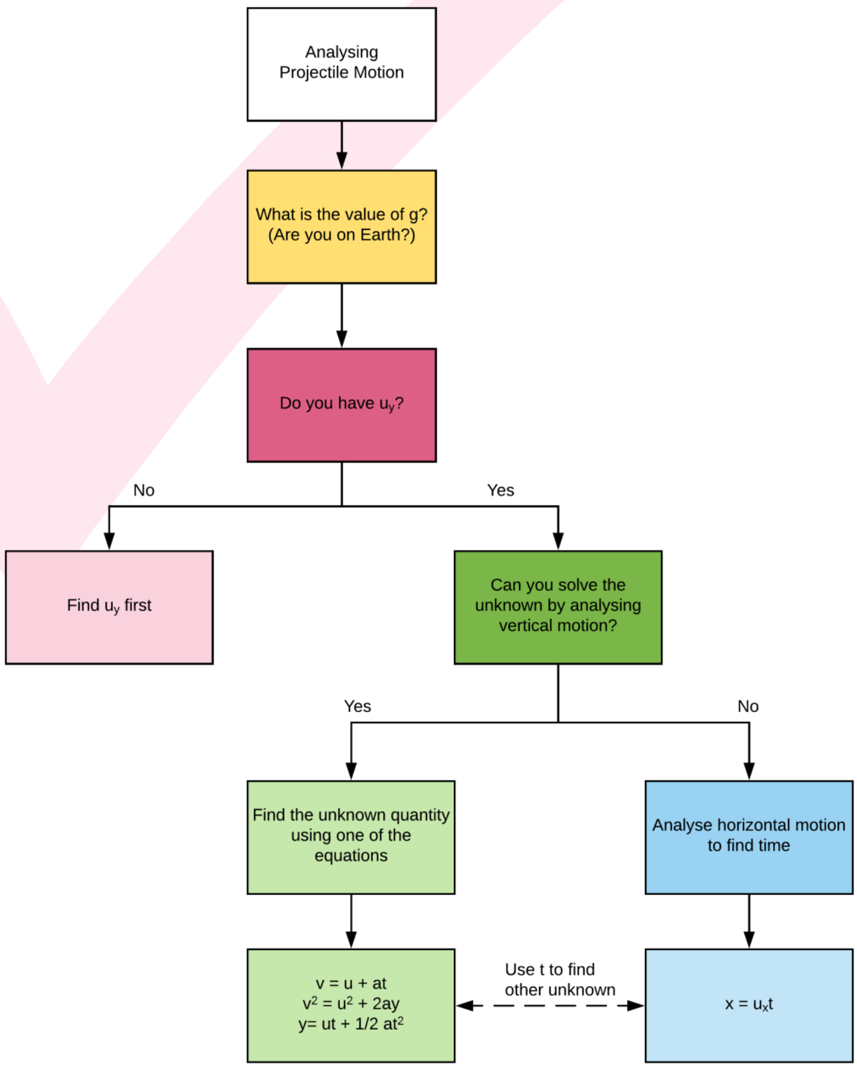


	Ball A	Ball B
Horizontal motion		
Vertical motion		
Shape of trajectory		

❒ Strategy for solving projectile motion

- Important information: Apply the following rules when solving projectile motion questions.
  - Vertical velocity is zero $(v_y = 0)$ at the maximum height because the projectile stops moving up and begins to fall down.
  - By convention, **up** is defined as **positive** and **down** is defined as **negative**.
  - Acceleration due to gravity is equal to $\mathbf{-9.8m/s^2}$ (for problems on Earth) since up is defined as positive.
  - "Released from rest" or "dropped" corresponds to $u_y = 0$.

- When considering projectile motion problems, you should consider this **general approach:** (This may not apply to all situations so first check what is being asked in the question!)



Concept Check 1.1

Consider an object moving in the Earth's gravitational field.

(a) What are the forces acting

(i) Vertically[2] **1**

(ii) Horizontally[3] **1**

(b) What is the acceleration of the object

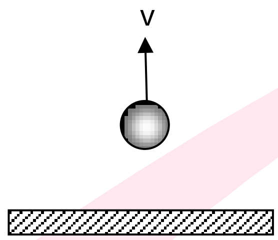
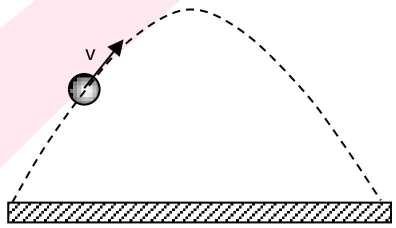
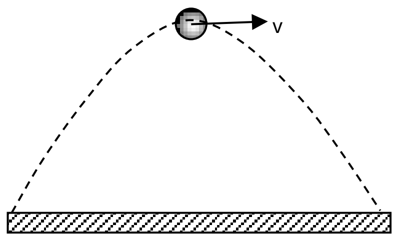
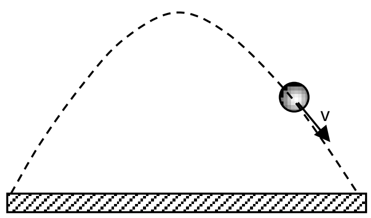
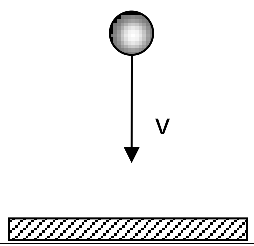
(i) Vertically[4] **1**

(ii) Horizontally[5] **1**

❐ Velocity and acceleration vectors for projectile motion

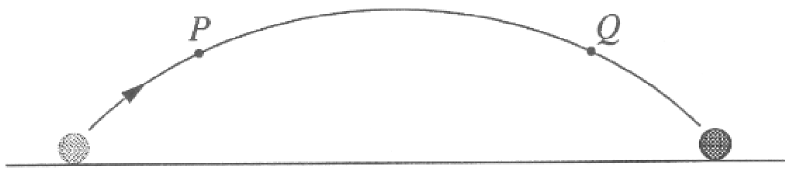
- There is **no fixed relationship** between the direction of the **velocity** vector and that of the **acceleration** vector for a moving particle. The figure below shows velocity vectors of a projectile for various motions.

- On the diagram, **indicate the acceleration vectors** of the projectile for its various motions.

Projectile thrown upward	
Rise of a projectile	
Projectile at top of trajectory	
Fall of a projectile	
Projectile thrown downward	

Concept Check 1.2 [HSC modified]

The diagram shows the trajectory of a golf ball.

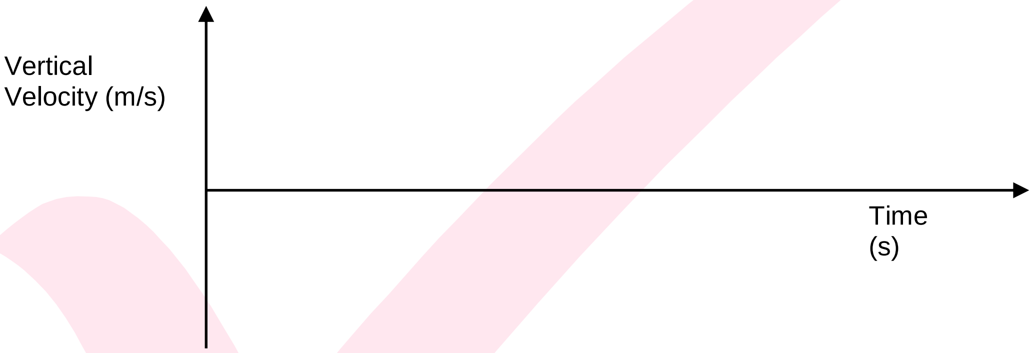


(a) Using arrows, show the direction of the acceleration of the ball at points P and Q.

2

(b) Sketch vertical velocity–time and acceleration–time graphs for this motion below.

2



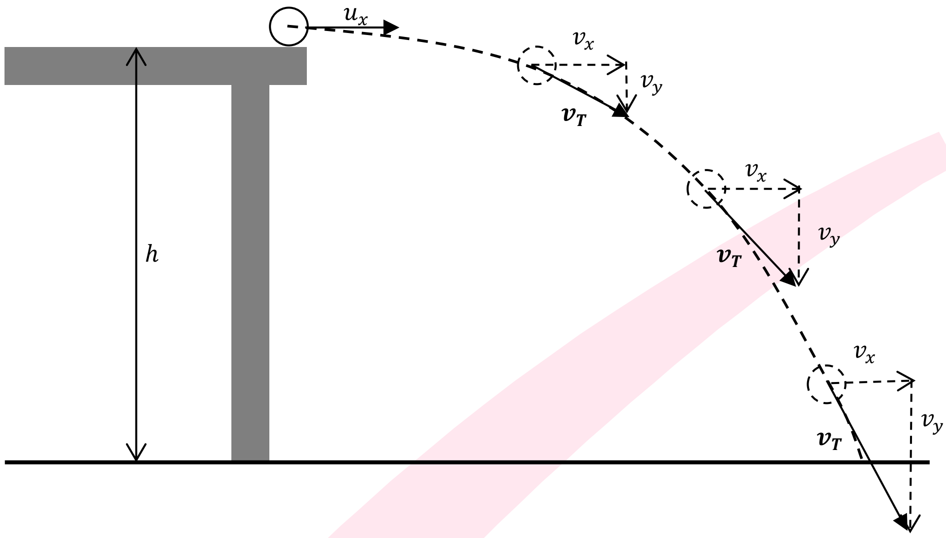
DID YOU KNOW?

Many students find these types of questions challenging. Such questions are often asked in exams as multiple choice.

2. Types of Projectile Motion

❒ Projectile motion: Type 1

- An object is rolled horizontally from a height of h above the ground with an initial velocity of u.

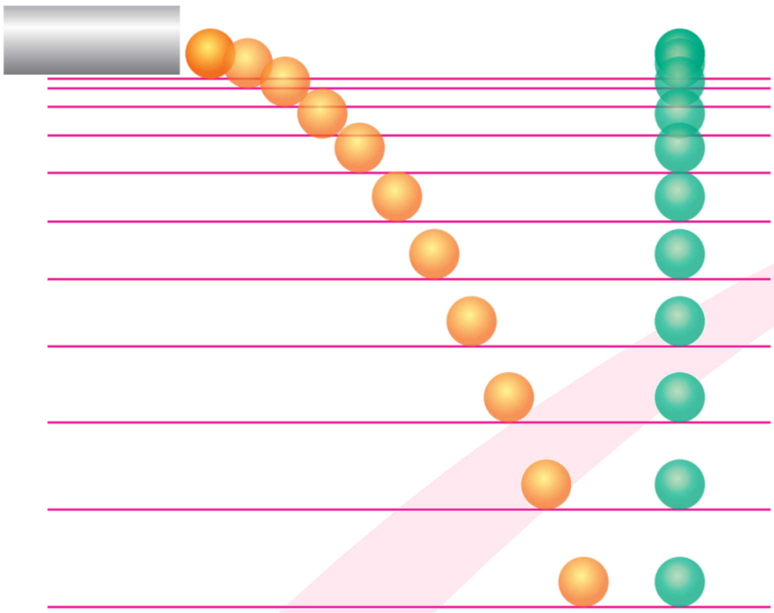


- **DEMONSTRATION:** (standalone Java) A projectile motion simulation which shows how the horizontal and vertical velocity changes during its flight.

- The table below outlines the important characteristics of a projectile rolled off a cliff or table.

Characteristic	Description
Velocity in the x–direction	Remains at its initial value
Velocity in the y–direction	**Initially zero** but increases as object falls

- The **stroboscopic image** below illustrates the falling motion of similar objects.
  - An orange ball is rolled horizontally and a green ball is dropped from rest from the same height.
  - The horizontal lines represent the fixed time interval between each image.



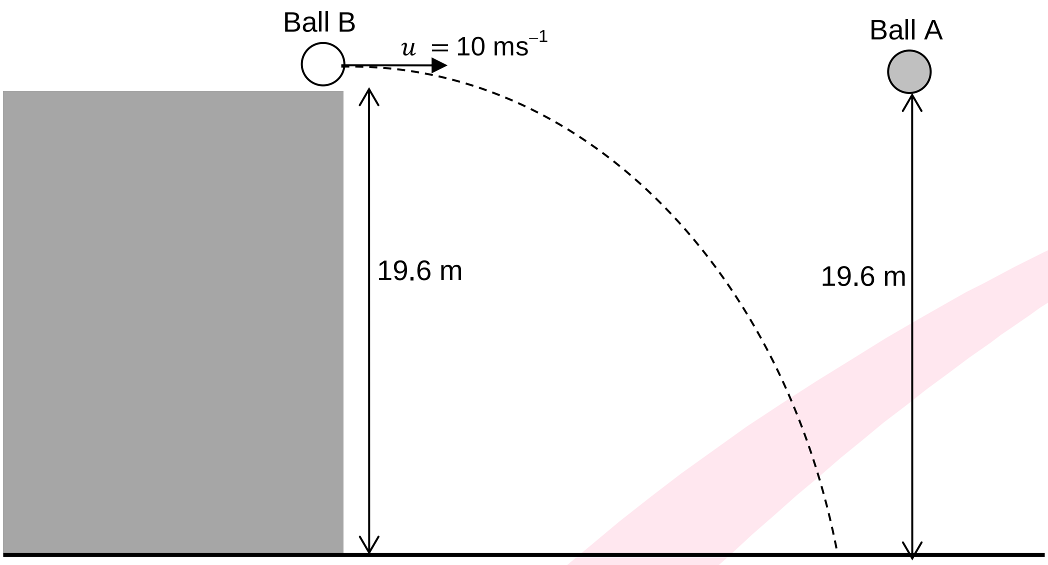
  - Compare the times taken for the balls to fall. Which ball reaches the floor first?[6]

  - Does the horizontal motion of the orange ball have any effect on the time taken to fall?[7]

- In projectile motion, **horizontal and vertical motion are independent** of each other. The only thing that the horizontal and vertical motion have in **common is time.**

Concept Check 2.1

Ball B is rolled with a speed of 10 ms^{-1} from the top of a building 19.6 m high while an identical ball A, is dropped vertically downwards from the same height.



Calculate:

(a) the time taken for **both** balls to reach the ground [8]

NOTE TO STUDENTS

The HSC exam marking committee expects students to use a methodical approach:

State the equation → substitute values in SI units → State answer in SI units

Ball A:

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

$-19.6 = 0 + \frac{1}{2}(-9.8)t^2$ (Note: the minus sign applies to Δy & a_y)

Ball B:

$$\Delta y = u_y t + \frac{1}{2} a_y t^2$$

Therefore the time taken for both balls to reach the ground is ____________.

DISCUSSION:

Why is the time of flight for ball A and B equal?

(b) the velocity the balls hit the ground with[9]

Ball A

$v_y = u_y + at$

Ball B

$v_y = u_y + at$

$v_x = u_x$

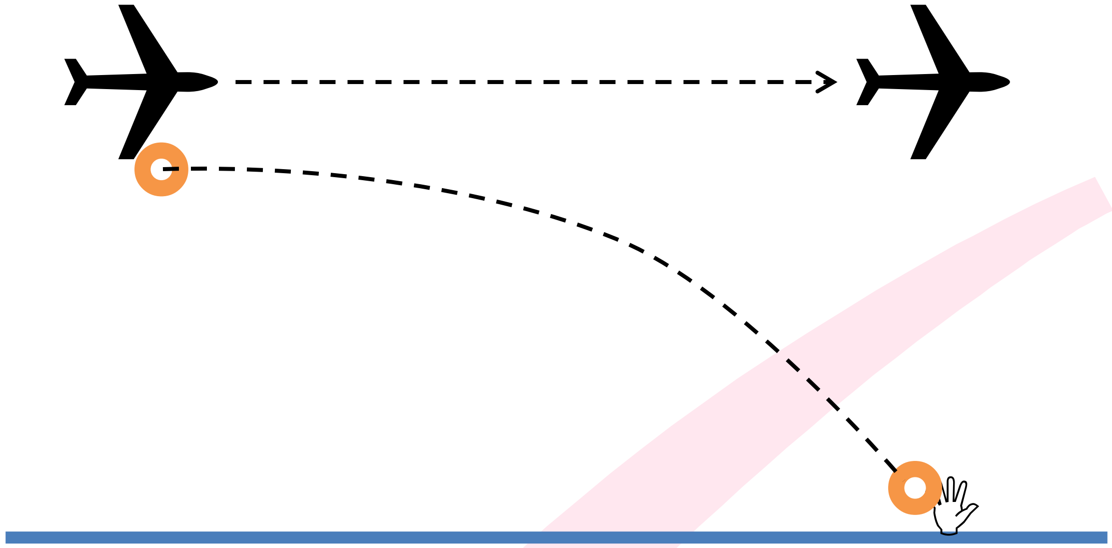
$v = \sqrt{(v_x)^2 + (v_y)^2}$

DID YOU KNOW?

The function *Pol(x,y)* on your Casio calculator can quickly determine the length of the hypotenuse and the angle in a right angled triangle.

Concept Check 2.2

A rescue plane is flying at a constant elevation of 1200 m with a speed of 360 kmh^{-1} towards a point directly over a person struggling in the water. The plane drops a rescue capsule and then continues in level flight as shown in the diagram. Ignore the effects of air resistance.



(a) The plane is directly above the capsule when it impacts with the water. Explain the relative positions of the plane and the rescue capsule.[10] **2**

(b) What is the time taken for the capsule to reach the person? [11] **2**

NOTE TO STUDENTS:

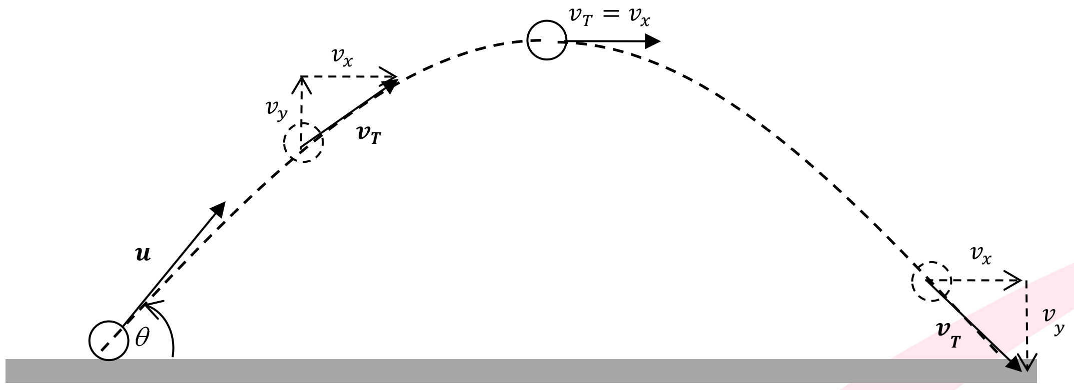
The plane is flying at a constant elevation (y) of 1200m, and so $u_y = 0$.

(c) At what horizontal distance, Δx, should the capsule be released if it is to reach the person?[12] **1**

(d) What is the velocity of the capsule upon impact?[13] **2**

❒ Projectile motion: Type 2

- The figure below shows a projectile fired from level ground at an angle of θ with an initial velocity of u.



  – On the same figure, draw the path of the projectile calculated using a computer by taking **air resistance** into account.

  – **ANIMATION:** (gif) A projectile motion simulation which shows how the horizontal and vertical velocity changes during flight.

- The table below outlines important characteristics of a projectile launched from level ground at an angle.

Characteristics	Description
Initial horizontal velocity	$u_x = u\cos\theta$
Initial vertical velocity	$u_y = u\sin\theta$
Horizontal velocity at max. height	$v_x = u_x$
Vertical velocity at max. height	zero
Angle of projection for max. range	45 degrees

NOTE TO STUDENTS:

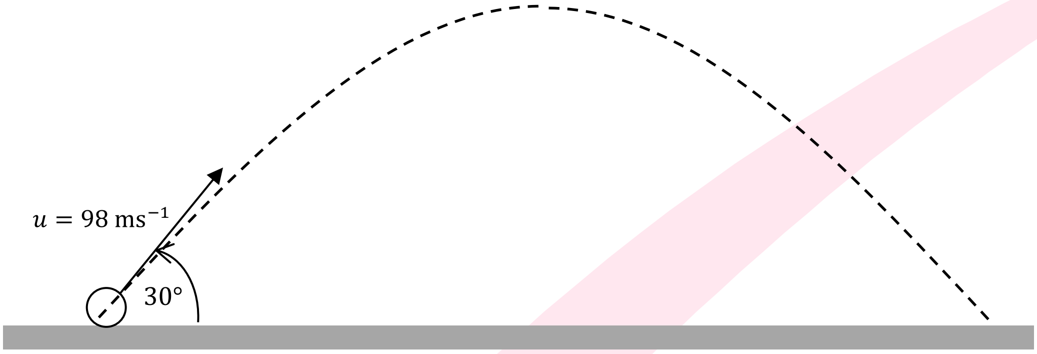
All of the different types of projectile motion use the same equations and mathematical techniques

Concept Check 2.3

A projectile is fired with an initial velocity of 98 m/s at an angle of 30° to the ground.

NOTE TO STUDENTS:

- Always sketch a diagram to assess its type and to visualise the situation.
- Always find u_y first regardless of whether asked or not.
- At maximum height, $v_y = 0$



Determine:

(a) the maximum height reached by the object [14] **1**

$v_y^2 = u_y^2 + 2a\Delta y$ (at maximum height, $v_y = 0$)

(b) the time it takes to reach the maximum height [15] **2**

$v_y = u_y + at$ (at maximum height, $v_y = 0$)

(c) the total time of the journey[16] **1**

(d) the range (horizontal distance travelled by the projectile)[17] **1**

$\Delta x = u_x t$

(e) the velocity it hits the ground with[18] **2**

NOTE TO STUDENTS: The velocity has horizontal **and** vertical components.

$v_y = u_y + at$

$v_x = u_x$

$v = \sqrt{(v_x)^2 + (v_y)^2}$

Concept Check 2.4 [HSC modified]

A projectile is fired at a velocity of 50 ms^{-1} at an angle of 26° to the horizontal.

Determine the range of the projectile.[19] **4**

COMMENTS FROM THE MARKING CENTRE
A substantial number of candidates were able to gain full marks for this question.
Most responses correctly resolved the initial velocity vector into its vertical and horizontal components. Better responses used the vector components in the relevant equations and identified the direction of the acceleration.

OUTCOMES ASSESSED: H6, H12

Criteria	Marks
• Correct solution given (working shown) • Correct substitutions (both data and signs) into relevant formulae (arithmetic errors ignored)	4
• Relevant formulae used but u_x and u_y transposed consistently OR • All correct but time of flight value is halved OR • Lack of use of sign convention	3

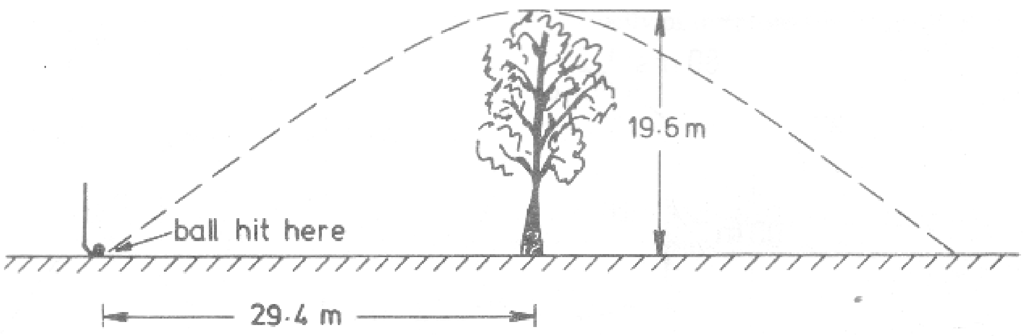
(a) Calculate horizontal and vertical components of the initial velocity **1**

(b) Find the time taken to reach maximum height **2**

(c) Hence calculate the range **1**

Concept Check 2.5 [HSC]

A golfer hits a golf ball which is 29.4 m in front of a large tree. The tree is 19.6 m high.



Take g, the acceleration due to gravity to be 9.8 m/s^2.

(a) With what vertical component of velocity must the golfer hit the ball so that it will just pass over the top of the tree? Show your working.[20] **2**

(b) How long will it take the ball to travel the horizontal distance of 29.4m? Show your working.[21] **2**

(c) At what angle to the horizontal did the golfer hit the ball? Show your working.[22] **2**