ABOUT THIS BOOK

THIS BOOK IS COMPLIANT WITH THE BOSTES NSW MATHEMATICS STAGE 5.3 SYLLABUS.

Content in the **Year 9 Maths Max Series™ Volume 1** covers the topics **'Algebraic Techniques and Surds & Indices'** from the 'Number & Algebra' strand of the NSW Mathematics Stage 5.2 and 5.3 Syllabus.

STRAND	TOPICS	CONTENT
Number and Algebra	Algebraic Techniques	• Add and subtract algebraic fractions. • Expand binomial products. • Factorise quadratic expressions.
	Surds & Indices	• Define rational and irrational numbers. • Simplify expressions involving surds by addition, subtraction, multiplication, and division. • Rationalise the denominator of surds.

For detailed information including the contents of the topics, please visit the BOSTES website link available on https://www.matrix.edu.au/BOSTES

THIS BOOK IS NOT YOUR ORDINARY TEXTBOOK.

Are you constantly surprised to find your school exam questions differ from the textbook questions you spent hours perfecting? Do you find those practice papers your teacher provided for you didn't quite prepare you for the exam?

Textbooks provide course content in a way that helps students gain a sound knowledge and understanding of the fundamental concepts.

But this is insufficient to achieve exam success at school as students are required to:

- demonstrate *extensive* knowledge and skills
- apply maths concepts to unfamiliar questions/situations
- use sophisticated multi-step reasoning
- exhibit excellent problem solving skills

THIS IS THE LAST PLACE TO LOOK BEFORE YOUR EXAM.

The **Year 9 Maths Max Series™ Volume 1** is designed for **exam preparation**, as well as, providing students the opportunity to further develop and practice the fundamental concepts. When it is used correctly, students will gain a significant advantage in their school exams.

The **Year 9 Maths Max Series™ Volume 1** works by:

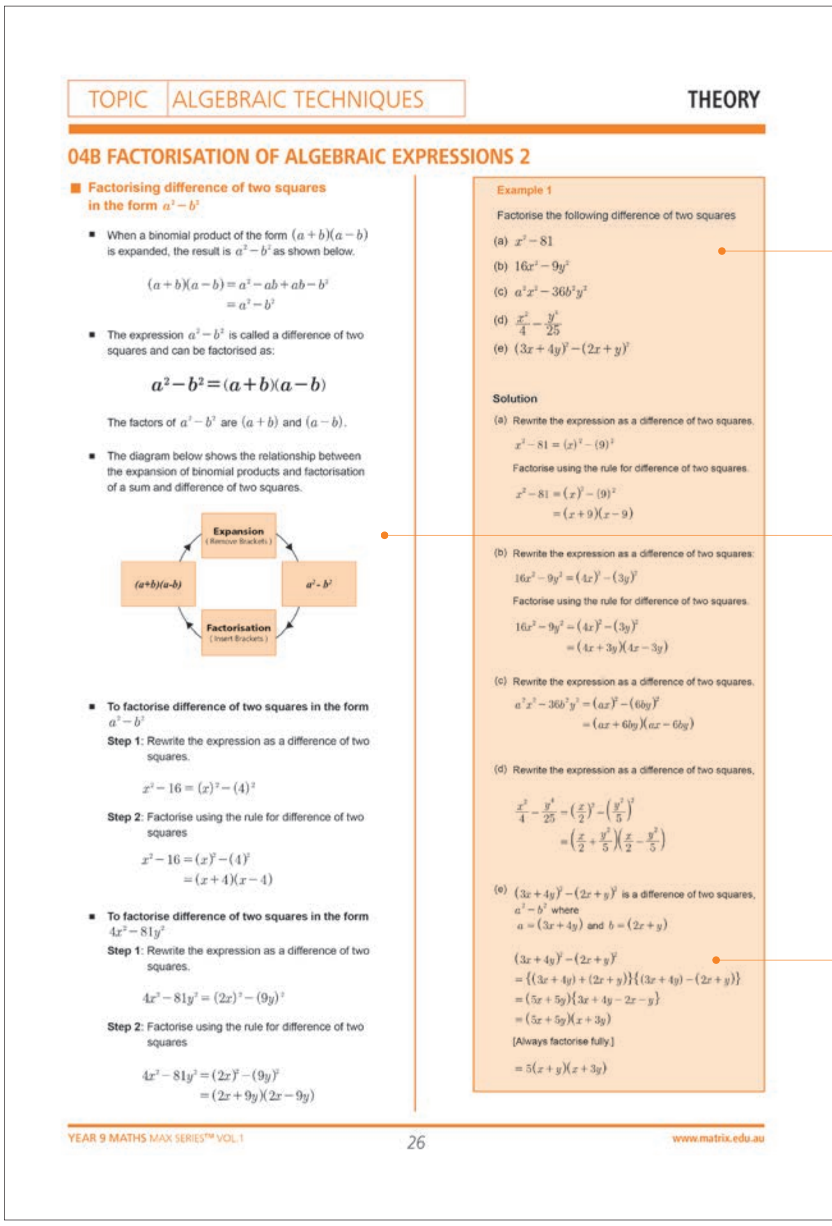
- exposing students to exam-style questions to reduce the chance of encountering unfamiliar problems.
- effectively preparing students for school exams by bridging the knowledge and skills gap between their textbooks and exam questions.
- improving students' problem solving skills by sharing the techniques not published in textbooks.

HOW TO USE THIS BOOK

Students should only use this book **after** gaining a sound knowledge and understanding of the key concepts of the mathematics course content. This book will effectively prepare students for school exams when the following steps are closely followed.

STEP 1 Review theory contents

At the beginning of each topic, a concise summary of the key concepts is provided. Be sure to read and understand it before attempting any questions.



TOPIC | ALGEBRAIC TECHNIQUES THEORY

04B FACTORISATION OF ALGEBRAIC EXPRESSIONS 2

Factorising difference of two squares in the form a^2-b^2

- When a binomial product of the form $(a+b)(a-b)$ is expanded, the result is a^2-b^2 as shown below.

$$\begin{aligned}(a+b)(a-b)&=a^2-ab+ab-b^2\\&=a^2-b^2\end{aligned}$$

- The expression a^2-b^2 is called a difference of two squares and can be factorised as:

$$a^2-b^2=(a+b)(a-b)$$

The factors of a^2-b^2 are $(a+b)$ and $(a-b)$.

- The diagram below shows the relationship between the expansion of binomial products and factorisation of a sum and difference of two squares.

Expansion (Remove Brackets)
$(a+b)(a-b)$
a^2-b^2
Factorisation (Insert Brackets)

- **To factorise difference of two squares in the form a^2-b^2**

Step 1: Rewrite the expression as a difference of two squares.

$$x^2-16=(x)^2-(4)^2$$

Step 2: Factorise using the rule for difference of two squares

$$\begin{aligned}x^2-16&=(x)^2-(4)^2\\&=(x+4)(x-4)\end{aligned}$$

- **To factorise difference of two squares in the form $4x^2-81y^2$**

Step 1: Rewrite the expression as a difference of two squares.

$$4x^2-81y^2=(2x)^2-(9y)^2$$

Step 2: Factorise using the rule for difference of two squares

$$\begin{aligned}4x^2-81y^2&=(2x)^2-(9y)^2\\&=(2x+9y)(2x-9y)\end{aligned}$$

Example 1

Factorise the following difference of two squares

(a) x^2-81

(b) $16x^2-9y^2$

(c) $a^2x^2-36b^2y^2$

(d) $\frac{x^2}{4}-\frac{y^4}{25}$

(e) $(3x+4y)^2-(2x+y)^2$

Solution

(a) Rewrite the expression as a difference of two squares.

$$x^2-81=(x)^2-(9)^2$$

Factorise using the rule for difference of two squares.

$$\begin{aligned}x^2-81&=(x)^2-(9)^2\\&=(x+9)(x-9)\end{aligned}$$

(b) Rewrite the expression as a difference of two squares:

$$16x^2-9y^2=(4x)^2-(3y)^2$$

Factorise using the rule for difference of two squares.

$$\begin{aligned}16x^2-9y^2&=(4x)^2-(3y)^2\\&=(4x+3y)(4x-3y)\end{aligned}$$

(c) Rewrite the expression as a difference of two squares.

$$\begin{aligned}a^2x^2-36b^2y^2&=(ax)^2-(6by)^2\\&=(ax+6by)(ax-6by)\end{aligned}$$

(d) Rewrite the expression as a difference of two squares.

$$\begin{aligned}\frac{x^2}{4}-\frac{y^4}{25}&=\left(\frac{x}{2}\right)^2-\left(\frac{y^2}{5}\right)^2\\&=\left(\frac{x}{2}+\frac{y^2}{5}\right)\left(\frac{x}{2}-\frac{y^2}{5}\right)\end{aligned}$$

(e) $(3x+4y)^2-(2x+y)^2$ is a difference of two squares, a^2-b^2 where $a=(3x+4y)$ and $b=(2x+y)$

$$\begin{aligned}&(3x+4y)^2-(2x+y)^2\\&=\{(3x+4y)+(2x+y)\}\{(3x+4y)-(2x+y)\}\\&=(5x+5y)\{3x+4y-2x-y\}\\&=(5x+5y)(x+3y)\end{aligned}$$

[Always factorise fully.]

$$=5(x+y)(x+3y)$$

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Various examples have been carefully curated to expose students to different exam-style questions.

A concise summary of the key concepts are provided at the beginning of each section.

A step-by-step worked solution is provided for each example to enhance your understanding.

STEP 2 Study the examples and its solutions to gain clarity

The examples expose students to various exam-style questions. The step-by-step worked solution for each example provides clear multi-step reasoning to enhance your understanding and provide a structure for maximising marks in exams.

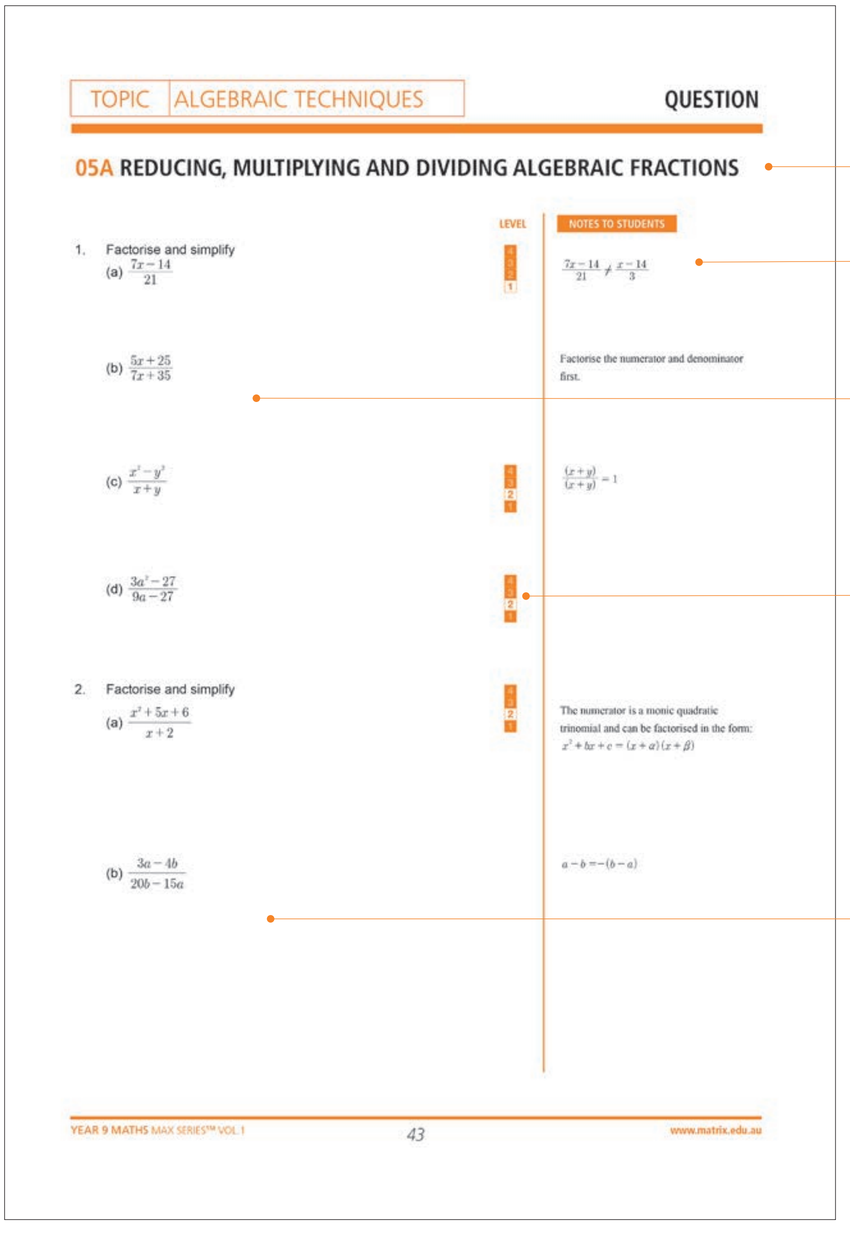
STEP 3 Attempt practice questions to improve problem solving skills

Questions are organised in a progressive manner to facilitate independent learning.

If you have difficulty, the **'NOTES TO STUDENTS'** section provides you with hints to encourage problem solving without referring to the solution.

The *common errors* section helps you remember what not to do in exams.

The **'LEVEL'** of difficulty is a useful tool to assess your ability to solve new, unseen and difficult problems.


TOPIC ALGEBRAIC TECHNIQUES

QUESTION

05A REDUCING, MULTIPLYING AND DIVIDING ALGEBRAIC FRACTIONS

LEVEL | NOTES TO STUDENTS

1. Factorise and simplify
   (a) $\frac{7x-14}{21}$ — $\frac{7x-14}{21} \neq \frac{x-14}{3}$
   (b) $\frac{5x+25}{7x+35}$ — Factorise the numerator and denominator first.
   (c) $\frac{x^2-y^2}{x+y}$ — $\frac{(x+y)}{(x+y)}=1$
   (d) $\frac{3a^2-27}{9a-27}$

2. Factorise and simplify
   (a) $\frac{x^2+5x+6}{x+2}$ — The numerator is a monic quadratic trinomial and can be factorised in the form: $x^2+bx+c=(x+\alpha)(x+\beta)$
   (b) $\frac{3a-4b}{20b-15a}$ — $a-b=-(b-a)$

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Refer to the section title for the relevant Theory.

Students can use hints from 'NOTES TO STUDENTS' to solve a problem without viewing at the solution.

Questions are organised in a progressive manner to help students build their confidence.

Questions are categorised into 4 levels of difficulty:

1: Basic
2: Fundamental
3. Difficult
4. Challenging

Spacing provided is a good indication of the amount of working required.

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03 SPECIAL PRODUCTS

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- ☐ Different pronumeral denominators
- ☐ Quadratic denominators

01 SIMPLE ALGEBRAIC EXPRESSIONS

Expansions of the form: $a(b+c)$

- Parentheses or brackets are used to group terms together and imply multiplication.
- To expand an algebraic expression with a bracket, multiply each term in the bracket by the term immediately outside the bracket, taking care with negative terms.

$$a(b+c)=ab+ac$$
$$a(b-c)=ab-ac$$
$$-a(b+c)=-ab-ac$$
$$-a(b-c)=-ab+ac$$

- After expanding, simplify the expression by collecting like terms.

Example 1

Expand the following

(a) $4(2x+3y)$

(b) $x(3x-5)$

(c) $x-(5y-1)$

(d) $2(x+y+1)$

(e) $-(x+y-z)$

Solution

(a) $4(2x+3y)=4\times 2x+4\times 3y=8x+12y$

(b) $x(3x-5)=3x^2-5x$

(c) $x-(5y-1)=x-5y+1$

[Note: $-(a-b)=-a+b$]

(d) $2(x+y+1)=2x+2y+2$

(e) $-(x+y-z)=-x-y+z$

[Note: $-(a+b)=-a-b$]

Example 2

Expand and simplify the following algebraic expressions

(a) $4(2a-3b)+3(2b+4a)$

(b) $x-(3x-5)$

(c) $3x(2x-7)-5(2x-7)$

Solution

(a) $4(2a-3b)+3(2b+4a)$

[Remove the brackets]

$=8a-12b+6b+12a$

[Collect like terms and simplify]

$=20a-6b$

[Express in the alphabetical order]

(b) $x-(3x-5)$

[Note: $-(3x-5)=-1\times(3x-5)$]

$=x-3x+5$

$=-2x+5$

(c) $3x(2x-7)-5(2x-7)$

[Remove the brackets. Be careful of the product of negative signs]

$=6x^2-21x-10x+35$

$=6x^2-31x+35$

Expansions of expressions with multiple brackets

- To expand algebraic expressions with multiple sets of brackets, always begin with the innermost brackets.

Example

Expand and simplify the following

$2\{5a^2+3(2a-3a^2)\}$

Solution

$2\{5a^2+3(2a-3a^2)\}$

[Remove the innermost brackets first]

$=2\{5a^2+6a-9a^2\}$

[Collect like terms in the bracket]

$=2\{-4a^2+6a\}$

[Remove the remaining brackets]

$=-8a^2+12a$

[Collect like terms and simplify]

01 SIMPLE ALGEBRAIC EXPRESSIONS

LEVEL

NOTES TO STUDENTS

1. Expand the following

4 3 2 1

(a) $3(x+2)$

$a(b+c)=ab+ac$

(b) $5(2-5x)$

$a(b-c)=ab-ac$

(c) $3a(a-2b+1)$

Remember to multiply every term inside the bracket by $3a$.

2. Expand and simplify

4 3 2 1

(a) $3x-5(x-7)$

Be careful with signs when a negative number is involved in multiplication:

$-\times+=-$

$-\times-=+$

(b) $3x(2x+y)+y(3x-5y)$

Write algebraic expressions with multiple pronumerals in alphabetical order:

$y\times 3x=3xy$

3. Expand and simplify

4 3 2 1

(a) $7a(2a-3b+4c)$

$-a(b-c+d)=-ab+ac-ad$

(b) $2a(a-2b+3)-a(-3a-4b+5)$

$-a(-b-c+d)=+ab+ac-ad$

(c) $5a(2a-3b+c)-4b(a-2b+3c)-c(2a+5b-c)$

01 SIMPLE ALGEBRAIC EXPRESSIONS

LEVEL

NOTES TO STUDENTS

4. Expand

(a) $a^4(a^7 - a^5 + 1)$

4 3 2 1

$a^x \times a^y = a^{x+y}$

(b) $8a^2b(-3a^3b^2 + 4a^5b^6)$

(c) $-2xy^2(x^2y - 5xy^3)$

5. Expand and simplify

(a) $2a^2 - 8a - \{5a^2 + (2a - 3a^2)\}$

4 3 2 1

Work from the inside out by expanding the innermost set of brackets first.

(b) $3a - 4b - \{2a - 7b - (5a - 3b)\}$

01 SIMPLE ALGEBRAIC EXPRESSIONS

LEVEL 4 3 2 1

6. Expand and simplify

(a) $4\{2x-6y-(x+2y)\}-2\{5x-3(2x-7y)\}$

(b) $7x^2-3[x-2-\{5-3x+4x(2x-1)+9x\}]$

(c) $9x-[5x-3y-\{4x-(y-3x)\}]-2x+5y$

NOTES TO STUDENTS

Expand and simplify the expressions in the inner brackets first before expanding further.

$-(y-3x)=-y+3x$

02 BINOMIAL PRODUCTS

Binomial expressions and products

- A **binomial expression** is an expression with **two terms**. An example of a binomial expression is $x + 2y$.
- The **product of two binomial expressions** is called a **binomial product**. An example of a binomial product is $(x+2y)(3x-y)$.

The Distributive Law

- Binomial products can be expanded using the Distributive Law:

$$\begin{aligned}(ax+b)(cx+d) &= ax(cx+d)+b(cx+d)\\ &= acx^2+adx+bcx+bd\end{aligned}$$

Example

Use the Distributive Law to expand and simplify the following

(a) $(2x+9)(x+4)$

(b) $(5-2x)(3-x)$

(c) $(x+2)(x^2-2x+4)$

Solution

(a) $(2x+9)(x+4)$

$= 2x(x+4)+9(x+4)$

$= 2x^2+8x+9x+36$

$= 2x^2+17x+36$

(b) Be very careful when there is a negative sign before the bracket. Don't forget to change the signs of all the terms in the bracket when you expand.

$(5-2x)(3-x)$

$= 5(3-x)-2x(3-x)$

$= 15-5x-6x+2x^2$

$= 2x^2-11x+15$

(c) The second bracket has three terms. Multiply each term in the second bracket by x and then by $+2$. Then simplify by collecting like terms.

$(x+2)(x^2-2x+4)$

$= x(x^2-2x+4)+2(x^2-2x+4)$

$= x^3 \cancel{-2x^2} \cancel{+4x} \cancel{+2x^2} \cancel{-4x} + 8$

$= x^3+8$

The FOIL Method

- The Distributive Law method of expanding binomial products can be shortened into what is called the FOIL Method.
- **FOIL** stands for:

F - First in each bracket are multiplied

O - Outsides in each bracket are multiplied

I - Insides in each bracket are multiplied

L - Last in each bracket are multiplied

$$\begin{aligned}&(ax+b)(cx+d)\\ &= \overbrace{(ax\times cx)}^{F} + \overbrace{(ax\times d)}^{O} + \overbrace{(b\times cx)}^{I} + \overbrace{(b\times d)}^{L}\end{aligned}$$

$$\begin{aligned}&(ax-b)(cx+d)\\ &= (ax\times cx)+(ax\times d)+(-b\times cx)+(-b\times d)\end{aligned}$$

Example

Use the FOIL Method to expand and simplify the following

(a) $(2x+9)(x+4)$

(b) $(5-2x)(3-x)$

(c) $(x+2)(x^2-2x+4)$

Solution

(a) $(2x+9)(x+4) = \overbrace{2x\times x}^{F} + \overbrace{2x\times 4}^{O} + \overbrace{9\times x}^{I} + \overbrace{9\times 4}^{L}$

$(2x+9)(x+4) = 2x^2+8x+9x+36$

$= 2x^2+17x+36$

(b) $(5-2x)(3-x)$

$= (5\times 3)+(5\times -x)+(-2x\times 3)+(-2x\times -x)$

$= 15-5x-6x+2x^2$

$= 2x^2-11x+15$

(c) $(x+2)(x^2-2x+4)$

$= (x\times x^2)+(x\times -2x)+(x\times 4)$

$\quad +(2\times x^2)+(2\times -2x)+(2\times 4)$

$= x^3-2x^2+4x+2x^2-4x+8$

$= x^3+8$

Which method, the Distributive Law or the FOIL Method, do you prefer?

Binomial product: $(x+a)(x+b)$

- Binomial products in the form of $(x+a)(x+b)$ can be expanded more efficiently if you use the rule below:

$$(x+a)(x+b)=x^2+(a+b)x+ab$$

- This rule can only be used if each bracket starts with the same term.

$$(p+a)(p+b)=p^2+(a+b)p+ab$$
$$(q+a)(q+b)=q^2+(a+b)q+ab$$
$$(x^2+a)(x^2+b)=(x^2)^2+(a+b)x^2+ab$$

- Study the solutions to the following expansions:

$$\begin{aligned}(x+3)(x+5)&=x^2+(3+5)x+(3\times 5)\\&=x^2+8x+15\end{aligned}$$

$$\begin{aligned}(x+9)(x-7)&=x^2+(9+-7)x+(9\times -7)\\&=x^2+2x-63\end{aligned}$$

$$\begin{aligned}(x-2)(x+11)&=x^2+(-2+11)x+(-2\times 11)\\&=x^2+9x-22\end{aligned}$$

$$\begin{aligned}(x-6)(x-5)&=x^2+(-6+-5)x+(-6\times -5)\\&=x^2-11x+30\end{aligned}$$

$$\begin{aligned}(x+7y)(x+11y)&=x^2+(7y+11y)x+(7y\times 11y)\\&=x^2+18xy+77y^2\end{aligned}$$

$$\begin{aligned}(x+9y)(x-12y)&=x^2+(9y+-12y)x+(9y\times -12y)\\&=x^2-3xy-108y^2\end{aligned}$$

$$\begin{aligned}(xy+8)(xy-3)&=(xy)^2+(8+-3)xy+(8\times -3)\\&=x^2y^2+5xy-24\end{aligned}$$

- Consider the numbers in the brackets, the coefficient of x and the constant term in the expansion. Do you see a connection between these values?

Example 1

Expand

$(p-4)(p+8)$

Solution

Since each bracket starts with p we can use the special product rule.

$$\begin{aligned}(p-4)(p+8)&=p^2+(-4+8)p+(-4\times 8)\\&=p^2+4p-32\end{aligned}$$

Example 2

Expand

$(m-8n)(m-9n)$

Solution

Since each bracket starts with m we can use the special product rule.

$$\begin{aligned}&(m-8n)(m-9n)\\&=m^2+(-8n-9n)m+(-8n\times -9n)\\&=m^2-17nm+72n^2\end{aligned}$$

[Express in the alphabetical order]

$$=m^2-17mn+72n^2$$

[Note: Always express pronumerals in the alphabetical order]

Example 3

Expand

$(x^2+5y)(x^2-12y)$

Solution

Since each bracket starts with x^2 we can use the special product rule.

$$\begin{aligned}&(x^2+5y)(x^2-12y)\\&=(x^2)^2+x^2(5y-12y)+(5y\times -12y)\\&=x^4-7yx^2-60y^2\\&=x^4-7x^2y-60y^2\end{aligned}$$

[Express in the alphabetical order]

02 BINOMIAL PRODUCTS

LEVEL

NOTES TO STUDENTS

1. Use the rule $(x+a)(x+b)=x^2+(a+b)x+ab$ to expand and simplify

(a) $(x+3)(x-7)$

Level: 4 3 2 **1**

$(x+a)(x-b)=x^2+(a-b)x-ab$

(b) $(x-3)(x-8)$

(c) $(a-3bc)(a-5bc)$

Level: 4 3 **2** 1

Remember to express your answer in alphabetical order.

(d) $(mk+6)(mk-9)$

$(xy+a)(xy-b)=(xy)^2+(a-b)xy-ab$

2. Use the FOIL Method to expand and simplify

(a) $(3x+1)(2x-1)$

Level: 4 3 **2** 1

$(a+b)(c-d)$
$=(a\times c)+(a\times -d)+(b\times c)+(b\times -d)$
$=ac-ad+bc-bd$

(b) $(2x-7y)(4x-9y)$

$(a-b)(c-d)$
$=(a\times c)+(a\times -d)+(-b\times c)+(-b\times -d)$
$=ac-ad-bc+bd$

(c) $(2p-5q)(3p-4q)$

3. Expand and simplify

$(7x-1)(2x^2-9x+5)$

Level: 4 **3** 2 1

$(a+b)(c+d+e)$
$=a(c+d+c)+b(c+d+e)$
$=ac+ad+ae+bc+bd+be$