1. Definition of a Polynomial

❒ What is a polynomial?

- A polynomial $P(x)$ is an algebraic expression of the form

Degree

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_3x^3 + a_2x^2 + a_1x + a_0$$

Leading term Leading coefficient Constant

where all powers of x **are integers greater than or equal to zero.**

– $a_0,\ a_1,\ a_2,\ \ldots\ ,\ a_n$ are called **coefficients**, a_n is called the **leading coefficient**.
– a_0 is called the **constant term**.
– a_nx^n is called the **leading term**.
– The **degree** refers to the highest power of x

- When $a_n = 1$, the polynomial is called a **monic polynomial**.

- $P(x)$ is a **constant polynomial** if $a_1 = a_2 = \ldots\ = a_n = 0$ and $P(x) = a_0$ (where a_0 is some constant).

- If all the co-efficients are zero, i.e. $a_0 = a_1 = a_2 = \ldots\ = a_n = 0$ then the polynomial $P(x) = 0$ is called a **zero polynomial** or a polynomial equation of degree 0.

- Real values of x that satisfy the equation $P(x) = 0$ are called the real **roots** or **zeroes** of the polynomial.

> **Note To Students:**
> When solving the **roots** or **zeroes** of a polynomial, this is also solving the x –intercepts of the polynomial. This will be useful when graphing the polynomial in Lesson 4.

Example 1

Consider the following polynomial.

$$P(x) = 3x^2 + 4x - 7$$

(i) Label the polynomial with the following terms.

(i) Leading coefficient [1]

(ii) Constant term [2]

(iii) Leading term [3]

(ii) What is the degree of the polynomial? [4]...

(iii) Is this a monic polynomial? Why or why not? [5]

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Example 2

Identify the leading coefficient for each of the following polynomials and determine if it is monic or non- monic.

(a) $P(x) = x + 1$

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(b) $P(x) = 3 - 6x + 9x^2 - 12x^3$

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(c) $P(x) = x^6 - x^9 + 1$

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(d) $P(x) = 5x^2 - 10x + 15$

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Concept Check 1.1

(a) Consider the polynomial $P(x) = 2x^4 + 4x^3 - 2x^2 + 5x - 6$.

Complete the following: [6]

(iv) The degree of the polynomial is …………………..

(v) The leading term is ………………………………..…..

(vi) The leading coefficient is …………………………..

(vii) The constant term is ……………..……………………

(viii) The coefficient of x^2 is ……………………………..

(ix) The polynomial has ……………..……….. terms.

(b) Write down a monic polynomial of degree 3 with a constant term of 5. [7]

NOTE TO STUDENTS

Monic means that the leading coefficient is 1.

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(c) Explain why $P(x) = 5x^2 - 3x^5 + 4x + \frac{5}{x} - 1$ is not a polynomial. [8]

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(d) State the degree, leading coefficient and constant term of $P(x) = (x^2 + 2)(x^4 - 3) - x^6$. [9]

NOTE TO STUDENTS

You must expand first and remember to always write your polynomials with the highest power at the front down to the lowest power at the end.

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Concept Check 1.2

State whether or not the following algebraic expressions is a polynomial. If it is a polynomial, state the degree. If it is not a polynomial, give a reason.

(a) $5x^3 - 7x^{\frac{1}{2}} - 2$ [10]

(b) $\frac{x^2+3}{4}$ [11]

(c) $\frac{2}{3x^2+10}$ [12]

(d) $\sqrt{2}x + 3$ [13]

Concept Check 1.3

Write the following polynomials in the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_1 x + a_0$ and hence state:

(i) The degree.

(ii) The constant term.

(iii) The coefficient of the x^3 term.

(iv) The leading term.

(v) Whether or not the polynomial is monic.

(a) $P(x) = x^2 + 2x^3 + 8 - 7x$ [14]

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(b) $P(x) = (2x + 3)(x^2 - 4)$ [15]

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(c) $P(x) = x^3(8x + 1) + 7x - 11 - (2x^2 - 1)(4x^2 - 3)$ [16]

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❒ The language of polynomials

- The notation $P(x)$ is the same as $f(x)$ from the 'Functions and Relations' topic from last term. $P(x)$ is a polynomial with x as the **variable** of the polynomial. When evaluating:
  - $\boldsymbol{P(2)}$, **substitute the value of 2 in place of all $\boldsymbol{x}$ values**.
  - $\boldsymbol{P(a)}$, **substitute the value of $\boldsymbol{a}$ in place of all $\boldsymbol{x}$ values**.
  - $\boldsymbol{P(a+1)}$, **substitute the value of $\boldsymbol{(a+1)}$ in place of all $\boldsymbol{x}$ values**.

Example 1

Consider the polynomial $P(x) = x^3 + 2x^2 - 6x + 1$. Evaluate $P(2)$.

Solution

To find the value of $P(2)$, substitute $x = 2$ into $P(x)$.

$P(2) = (2)^3 + 2(2)^2 - 6(2) + 1 = 5$

Therefore $P(2) = 5$.

Example 2

Given $P(x) = -3x^3 + 2x + 9$, evaluate the following.

(a) $P(-1)$. [17]

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(b) $P(m)$. [18]

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(c) $P(x-1)$. [19]

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Concept Check 1.4

(a) Given $P(x) = x^3 - 3x^2 - x + 1$, evaluate $P(-2)$, $P(1)$ and $P(3)$. [20]

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(b) Given $P(x) = -x^4 - 2x^3 + x^2 - 10$, evaluate $P(3a)$ and $P\left(\frac{x}{3}\right)$. [21]

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2. Operations with Polynomials

❒ Addition and subtraction

- The sum or difference of two polynomials is found by collecting "like terms".

$$(a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0) \pm (b_n x^n + b_{n-1}x^{n-1} + \cdots + b_2x^2 + b_1x + b_0)$$
$$= (a_n \pm b_n)x^n + (a_{n-1} \pm b_{n-1})x^{n-1} + \cdots + (a_2 \pm b_2)x^2 + (a_1 \pm b_1)x + (a_0 \pm b_0)$$

- **To add or subtract polynomials:**
  (i) Find the **matching degree n terms** and
  (ii) Add or subtract their **coefficients**.

Example 1

Fully simplify the following polynomials.

(a) $P(x) + Q(x)$

$P(x) = x^3 - 3x^2 + 4x - 12$
$Q(x) = 7x^3 + 2x^2 - 3x + 8$

(b) $P(x) - Q(x)$

$P(x) = 5x^4 - 0x^3 - 9x^2 + 10x - 3$
$Q(x) = 2x^4 + 2x^3 - 10x^2 + 5x - 9$

Example 2

Consider the polynomials $P(x) = 3x^3 - x + 4$ and $Q(x) = x^3 + 2x^2 - 5x + 1$.

What is the sum of the two polynomials? [22]

Solution

$P(x) + Q(x) = (3x^3 - x + 4) + (x^3 + 2x^2 - 5x + 1)$
$= (3x^3 + x^3) + (2x^2) + (-x - 5x) + (4 + 1)$

$=$..

Note To Students:

Remember to write the polynomial in 'Degree' order when solving addition or subtraction of polynomials. This includes adding a zero- coefficient for terms that are not included.

e.g. $P(x) = x^2 - 3x^4 + 3x - 1$

Hence, $P(x) = -3x^4 + 0x^3 + x^2 + 3x - 1$

Concept Check 2.1

Consider the polynomials $P(x) = 2x^3 - 4x^2 - x + 3$ and $Q(x) = -x^3 + x^2 - 3x - 5$.

(a) Given $A(x) = P(x) - Q(x)$, find $A(x)$ and hence evaluate $A(3)$. [23]

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(b) Given $B(x) = 2[P(x) + Q(x)]$, find $B(x)$ and hence evaluate $B(-1)$. [24]

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