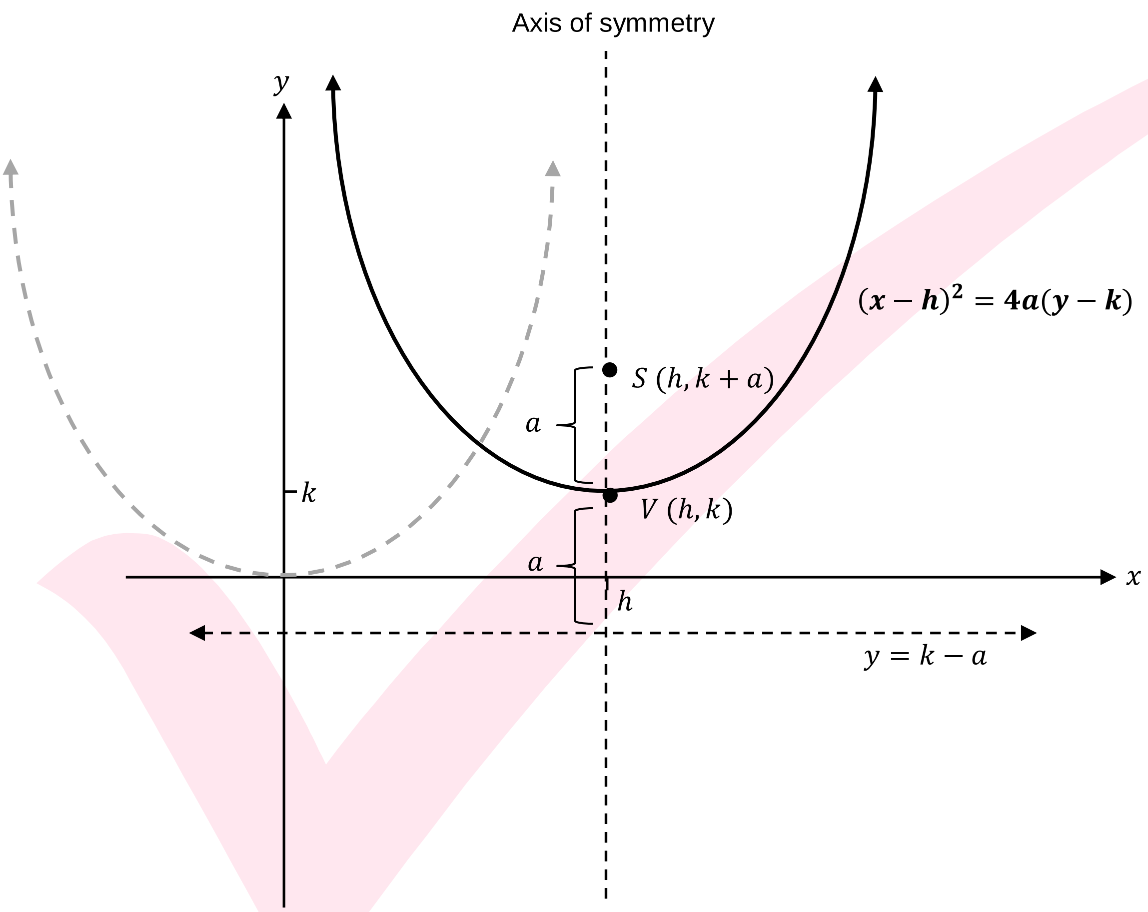
3. TRANSLATED PARABOLAS

❒ The Parabola with Vertex $V(h, k)$ and Axes Parallel to the y −axis

- Consider the concave up parabola with vertex $V(h, k)$ shown below. This parabola is obtained by translating the parabola $x^2 = 4ay$ h units horizontally and k units vertically.



– The vertex $V(h, k)$ lies midway between the focus and the directrix.

– The focal length is a.

– State the coordinates of the focus and the equation of the directrix. [42]

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– We can now re-write the equation with reference to the $x - y$ axes. The equation is $(x - h)^2 = 4a(y - k)$.

- The equation of a parabola with focus $S(h, k+a)$ and directrix $y = k - a$ is $(x-h)^2 = 4a(y-k)$.

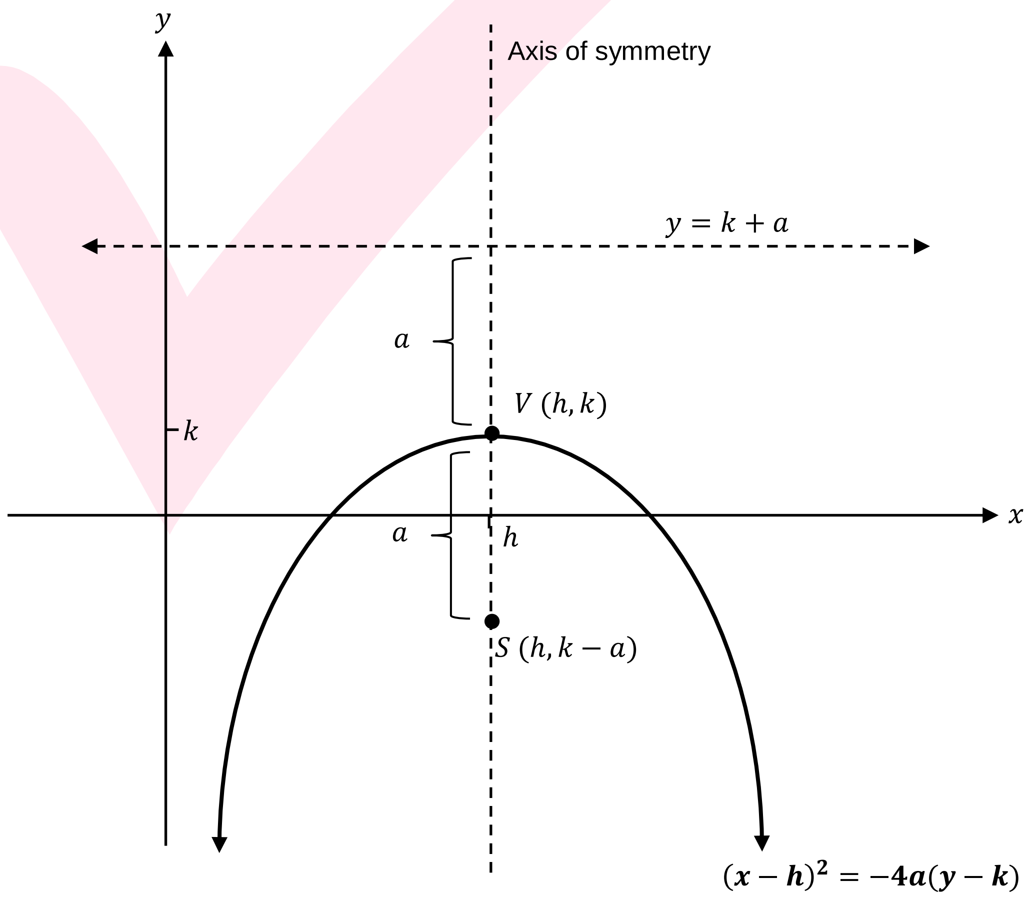
$$(\boldsymbol{x}-\boldsymbol{h})^2 = \boldsymbol{4a}(\boldsymbol{y}-\boldsymbol{k})$$

Equation of a concave up parabola with $V(h,k)$ $\rightarrow \cup$

- We can also conclude that the equation of a parabola with focus $S(h, k-a)$ and directrix $y = k + a$ is $(x-h)^2 = -4a(y-k)$. This is shown in the diagram below.

$$(\boldsymbol{x}-\boldsymbol{h})^2 = -\boldsymbol{4a}(\boldsymbol{y}-\boldsymbol{k})$$

Equation of a concave down parabola with $V(h,k)$ $\rightarrow \cap$



Example:

Find the equation of parabola $x^2 - 8x + 12y + 4 = 0$ and sketch, clearly identifying the focus, vertex and directrix.

Step 1: Complete the square, and express in the form $(x-h)^2 = -4a(y-k)$. [43]

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Step 2: Identify the focal length .

$a =$ ....[44]

Step 3: Hence, determine the vertex, focus and directrix. [45]

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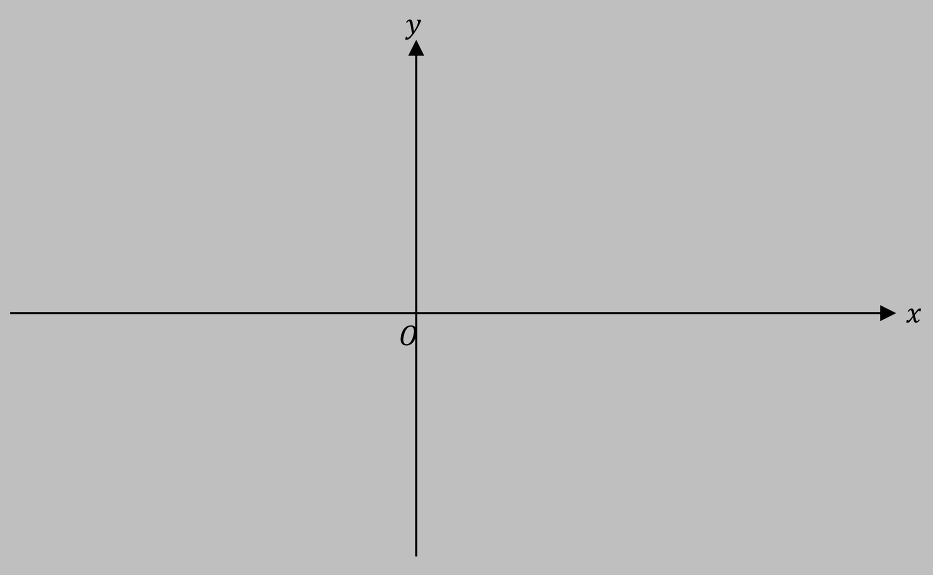
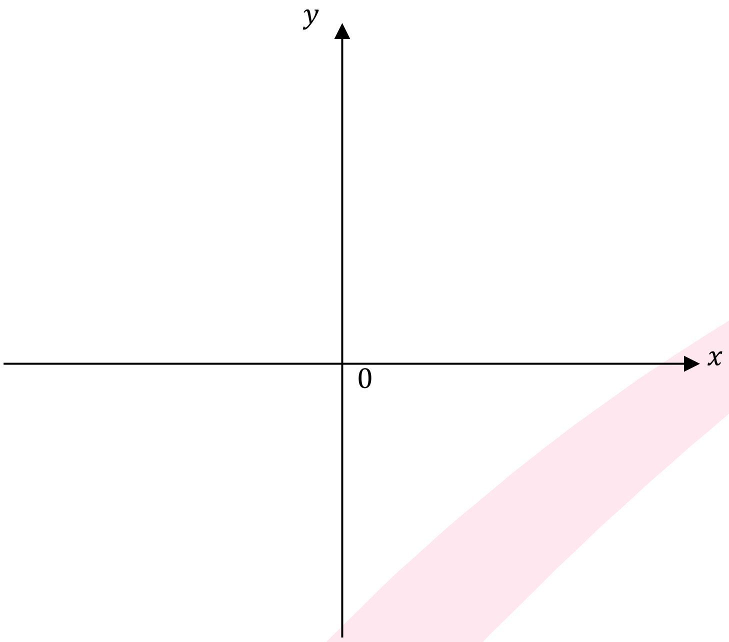
Step 4: Sketch the locus, showing all essential features.



Concept Check 3.1

(a) On the number plane below, plot the point $S(2,2)$ and draw the line $y=-4$. **1**



(b) Find the locus of a point which moves in a plane so that its distance from the point $S(2,2)$ is equal to its distance from the line $y=-4$. [46] **2**

Note to Students:
In this question, you are required to use the definition of a locus of a parabola to determine the equation of a parabola.

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Concept Check 3.2

Find the equation of the parabola with the following features:

(a) Focus $(2, -1)$ and directrix $y = -7$. [47] **2**

(b) Vertex $(2, -2)$, focal length 2 units, axis of symmetry $x = 2$ [48] **2**

(c) Focus $(2, -1)$ and vertex $(2, 2)$ [49] **2**

(d) Focus $(1, 7)$ and directrix $y = 3$ [50] **2**

(e) Vertex $\left(0, -\frac{3}{2}\right)$, directrix $y = \frac{1}{2}$ [51] **2**

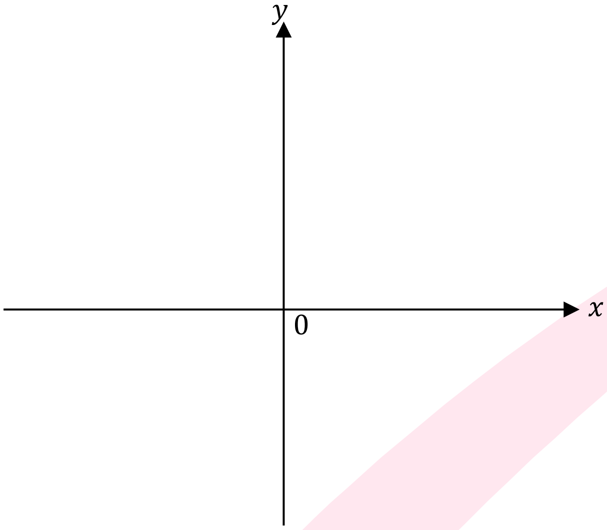
(f) Vertex $(-2, 3)$, with the axis of symmetry parallel to the y-axis, passing through the point $(-6, -5)$. [52] **2**

Note to Students:
Before determining the equation of the parabola, solve for the value of a.

Concept Check 3.3

A parabola has equation $x^2 = 8(4 - y)$.

(a) Draw a neat sketch of the parabola and clearly indicate on it the equation of its directrix, the coordinates of its focus and the coordinates of all points of intersection of the parabola with the coordinate axes. **1**



(b) Another parabola with equation $x^2 = 8y$ cuts the parabola $x^2 = 8(4 - y)$ at A and B. Find the coordinates of A and B. [53] **2**

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Concept Check 3.4

A parabola has equation $y = x^2 - 4x + 5$.

(a) Find the coordinates of its vertex and focus. [54] **2**

> **Note to Students:**
>
> Express in the form $(x - h)^2 = 4a(y - k)$ by completing the square.

(b) Find the equations of its directrix and latus rectum. [55] **1**

(c) Sketch the parabola, showing all essential features. **1**