1. **OVERVIEW OF TRIGONOMETRIC FUNCTIONS 1**

- **Introduction to Radians and Trigonometric Functions**

  - An interesting question to consider is “Why are there 360 degrees in a complete revolution”? Why not 390 degrees or 100 degrees? The remarkable answer is that this unit of measure springs from the ancient Babylonians. The Babylonians worked in base 60 rather than base 10 and used a calendar where the year was divided into 360 days. So a “degree” is just a “day”!

  - Not surprisingly, this unit of angular measure does not work well for higher levels of mathematical analysis and is particularly clumsy when dealing with the calculus. Our first lesson in term 2 will define and investigate a totally new way of quantifying angles called **radian** measure.

  - We will:
    - Define radian measure.
    - Discuss how we can convert from degrees to radians and vice versa.
    - Learn how to use radians on the calculator.
    - Establish some radian based formulae for areas and arc lengths of segments and sectors in circles.
You already have at your disposal a myriad of mathematical functions, ranging from the simple polynomials to the more advanced exponential and log functions. The introduction of radian measure opens up the door for the definition and development of the trigonometric functions. This lesson closes with a discussion of how to graph trigonometric functions and in the following book we will begin the analysis of their derivatives and integrals.

The calculus of the trigonometric functions forms the backbone of many of the later Extension 1 topics, in particular projectile and simple harmonic motion. Ensure that you are familiar with all of the trigonometric graphs and their properties as they will be used extensively in the rest of your H.S.C. mathematical studies.
2. **CIRCULAR MEASURE OF ANGLES**

- **Definition of Radian Measure**
  - A radian is the angle subtended at the centre of a unit circle by an arc of unit length.
  
  ![Diagram of unit circle and radian measure](image)

  - Here, \( \angle POQ = 1 \) radian.
  - It is the angle subtended at the centre of a unit circle by the arc \( PQ \) which measures 1 unit.

**Note to Students:**
1 Radian is the angle subtended by a unit arc on a unit circle. Note that 1\(^{c}\) \(\approx\) 60\(^\circ\), so one radian is quite a large angle.

- The following unit circle has an angle subtended at the centre by an arc \( PQ \) measuring 2 units

![Diagram of unit circle with angle subtended by arc](image)

- What is \( \angle POQ \) in radians? \( \ldots \ldots \ldots \ldots \) \(^{[1]}\)
Radian measure also works for circles of any radius.

- The following circle has a radius of 2 units and the arc $PQ$ measures 2 units.
  What is the size of $\angle POQ$ in radians? .......................... [2]

![Diagram of a circle with a radian measure]

- Definition of $\pi$ Radians

  ![Diagram of a circle with $\pi$ radians]

  The radius $OA$ of the circle is 1 unit.

  - The size of the angle $\angle AOB$ in degrees = ..........................[3]
  - The length of the arc $AB$ is approximately 3.14159265… or a number called $\pi$
  - Hence, $180^\circ$ = ...............[4] radians
Converting angles from degrees to radians:

- 180° is equivalent to $\pi$ radian
- $180° = \pi$ radians
- $1° = \frac{\pi}{180}$ radians
- $\therefore x° = \frac{\pi}{180} \times x$ radians

\[
x° = \frac{\pi}{180} \times x \text{ radians}
\]

- Hence $30° = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$ radians

Did you know:

$\pi° = 180°$ is the fundamental conversion factor between radians and degrees, just like $2.54\text{ cm} = 1\text{ inch}$.

Concept Check 2.1

Convert the following angles to radians:

(a) $60°$ leaving your answer in terms of $\pi$. [9]

(b) $55°$ correct to 4 decimal places. [6]

(c) $2°30'$ (Note $2°30' = 2 \cdot 5°$) [7]

(d) $405°$ in terms of $\pi$. [8]
Converting angles from radians to degrees:

- \( \pi \) radians is equivalent to 180°
- Rearranging the previous equation,
- \( \frac{\pi}{180} \times x \) radians = \( x^\circ \)
- \( \therefore \) \( x \) radians = \( \frac{180}{\pi} \times x^\circ \)

\[ x \text{ radians} = \frac{180}{\pi} \times x \text{ degrees}. \]

Hence 4 radians = \( \frac{180}{\pi} \times 4 \approx 229°11’ \) to the nearest minute.

Did you know:

- 60″ = 1’
- 60’ = 1° (Just like time!)
- The symbol for radians is ‘rad’ or the superscript ‘c’ e.g. \( \frac{\pi}{4} \text{ rad} \) or \( \frac{\pi}{4} \text{ c} \)

However note that the symbol is often omitted and is considered normal practise.

Concept Check 2.2

Convert the following angles from radian measure to degrees.

(a) \( \frac{5\pi}{18} \) °

(b) 2.25 radians to the nearest minute.

(c) \( \frac{2\pi}{7} \) to the nearest minute.

Discussion Question:

When do we use degrees and when do we use radians?
3. **ARC LENGTH, AREAS OF SECTORS & SEGMENTS**

- **Arc Length – Investigation**
  - To find the formula of the arc length, let's investigate a full circle.

  ![Diagram of a full circle with labels and angles](image)

  - From our previous knowledge we should understand that a full circle's arc length is the circumference = \(2\pi r\). The angle 'subtended' by the full circle is \(2\pi\) radians.

  - What about a quarter of a circle?

  ![Diagram of a quarter circle](image)

  - Arc length \(AB = \text{Quarter of full circle} = \ldots\)
  - \(\angle AOB = \text{Quarter of full circle} = \ldots\)
Arc Length – Formula

- Clearly, the arc length and the angle subtended at the centre by the arc (in radians) are directly related.
- Below is an arc $AB$ subtending an angle, $\theta$ radians, at the centre of a circle of radius $r$ units.

Using ratios to compare a full circle to the above sector, we find that

$$\frac{l}{\theta} = \frac{2\pi r}{2\pi}$$

where $l$ is the length of the arc $AB$.

Hence the length of the arc $AB$ is given by:

$$l = r\theta$$

where $\theta$ is measured in radians

**Concept Check 3.1**

(a) An arc subtends an angle of $75^\circ$ at the centre of a circle of radius $10$ cm.

(i) Express $75^\circ$ in radians. $^{[13]}$

**Note to Students:**

It is very important to convert your angle into radians before you use the $l = r\theta$ formula.

(ii) Hence find the length of the arc to the nearest millimetre. $^{[14]}$
(b) An arc of length 15 cm in a circle of radius 10 cm subtends an angle $\theta$ at the centre of the circle. Find the size of $\theta$ to the nearest minute. [15]

(c) The pendulum of a clock is 15 cm long. It travels through an angle of $28^\circ$ on each swing. Find the distance that the end of the pendulum moves on each swing. Give your answer correct to 2 decimal places. [16]

(d) A car travels along a circular bend of radius 750 metres at a speed of 50 km/h.

(i) Find the distance the car travels in one minute. [17]

(ii) Find the size of $\theta$, the angle through which the car turns in one minute. Give your answer correct to the nearest degree. [18]
Area of Sector – Investigation

- Again let’s investigate the full circle to find a relation for the area of a sector

- We should already know that a full circle of radius $r$ with the angle $2\pi$ radians at the centre has an area of $\pi r^2 \text{ units}^2$

- What about a quarter of a circle?

- Area of sector $AOB = \text{Quarter of full circle} = \ldots$
- $\angle AOB = \text{Quarter of full circle} = \ldots$
Area of the sector - Formula

- Again, the sector and the angle subtended at the centre (in radians) are directly related.

![Diagram of a sector](image)

- The area of the sector $AOB$ is found by using the ratio of the areas:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

Hence the area of the sector $AOB$ is given by:

$$A = \frac{1}{2} r^2 \theta$$

where $\theta$ is in radians

Concept Check 3.2

(a) A sector subtends an angle of 25° at the centre of a circle of radius 8 cm. Find the area of the sector in terms of $\pi$. [19]

Note to Students:
It is absolutely crucial when using $A = \frac{1}{2} r^2 \theta$ and $l = r \theta$ to measure $\theta$ in radians.
(b) A sector of area of 64 cm$^2$ subtends an angle $\theta$ at the centre of a circle of radius 10 cm. Find the size of $\theta$ correct to the nearest minute. [20]

(c) The arc length of a sector 54 cm. The sector subtends an angle $\theta$ at the centre of a circle of radius 9 cm. Find the exact area of the sector. [21]
(d) A sector is cut out of a circular piece of filter paper of diameter 10 cm. The angle of the sector is $150^\circ$. The cut edges of the sector are then pulled together to form a cone.

(i) Find the radius of the base of the cone. [22]

Did you know:
The length of the arc is the circumference of the base of the cone.

(ii) Calculate the area of the curved surface of the cone. [23]

Did you know:
The curved surface of the cone is the area of the sector.
(e) The diagram represents a rectangular paddock 45m by 20m. It is fenced completely and a horse is tied to a corner post, P, with a rope 30 metres long.

![Diagram of a rectangular paddock with a horse tied to a corner post]

(i) Show that the size of $\angle QPR$, correct to the nearest minute, is $41^\circ 49'$.

(ii) The arc $QR$ and the fences $RS$, $SP$ and $PQ$ bound the total area over which the horse can graze within the paddock. Calculate this area correct to the nearest square metre. [24]
The area of a minor segment cut off by a chord

Definition of a segment:
- A segment is an area of a circle cut off by a chord or secant.
- There will be a minor & major segment unless the chord is a diameter in which case there will be 2 semi-circles!

The area of the minor segment bounded by the arc \( PQ \) and the chord \( PQ \) is equivalent to the difference between the area of the sector \( POQ \) and the area of \( \Delta POQ \).

As an equation, this can be represented as:

\[
A_{\text{segment}} = A_{\text{sector}} - A_{\Delta POQ}
\]

\[
A_{\text{segment}} = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta
\]

Area of segment \( = \frac{r^2}{2} (\theta - \sin \theta) \)

where \( \theta \) is in radians.

Make sure that your calculator is on radian mode.

Note to Students:
For the rest of your mathematical studies you must take extreme care when using the trig buttons on your calculator. Make sure that the mode on your calculator (degrees vs radians) is aligned with your expectations. You must be able to switch between the modes quickly. Each calculator is a little bit different.
Concept Check 3.3

(a) An arc of a circle of radius 12 cm subtends an angle of 60° at the centre of the circle. Find the area of the minor segment bounded by the arc and the chord joining the endpoints of the arc to the nearest square centimetre. [25]

(b) Find the area of the minor segment of a circle of radius 6 cm and cut off by a chord of length 10 cm correct to 2 decimal places. [26]
(c) Two circles of radii 3 cm and 4 cm have their centres 5 cm apart. Calculate the area common to both circles correct to 2 decimal places. [27]

Note to Students:
Consider the following steps:
- Complete the diagram by drawing the radii of each circle and labelling all the lengths that you know
You should see two triangles which have a special property that allows you to complete the question
4. **GRAPHS OF TRIGONOMETRIC FUNCTIONS**

- **Investigation of the graph of** \( y = \sin x \)

  - To visualise the graph, let's investigate the values of \( y = \sin x \) from \( 0 \leq x \leq 2\pi \).

<table>
<thead>
<tr>
<th>Change in ( x )</th>
<th>Change in ( y = \sin x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \to \frac{\pi}{2} )</td>
<td>( \ldots \to \ldots )</td>
</tr>
<tr>
<td>( \frac{\pi}{2} \to \pi )</td>
<td>( \ldots \to \ldots )</td>
</tr>
<tr>
<td>( \pi \to \frac{3\pi}{2} )</td>
<td>( \ldots \to \ldots )</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} \to 2\pi )</td>
<td>( \ldots \to \ldots )</td>
</tr>
</tbody>
</table>

- As shown above, the sine graph goes through a complete series of changes in the domain \( 0 \leq x \leq 2\pi \). Further increases in \( x \) simply repeat these changes at intervals of \( 2\pi \).

- Hence the graph of the sine curve is shown below:

- Functions that change in this manner are called *periodic functions* and the length of the interval of a complete sin curve is called the *period* of the function. The *period of sin \( x \) is therefore* \[28\]

- Remember from Year 11 Extension 1 Trigonometry, \( y = \sin x \) can only have values in between 1 and -1. Hence, the *range of the curve must be* \[29\]

- The *amplitude* of the curve is defined as the maximum distance from the mean level (centre) OR half the distance from the peak (max value) and trough (min value). The amplitude is always positive.

- Hence, the *amplitude of the sin graph is* \[30\]

\[
y = \sin x
\]

**Period:** \( \ldots \)

**Amplitude:** \( \ldots \)
The Graph of \( y = A \sin x \)

- The family of curves shown in the diagram above is the result of assigning successive values to \( A \).
- The period for each curve is \( 2\pi \). Hence the value of \( A \) does not affect the period.
- Recall from Functions, for \( y = A[f(x)] \), the function \( y = f(x) \) is stretched vertically by a factor of \( A \).
- Hence the value of \( A \) affects the range and the amplitude of the curve.

\[
y = A \sin x
\]

- Period: \( 2\pi \)
- Amplitude: \( A \)

Did you know:
keep in mind \( 2\pi^C = 360^\circ \)
Concept Check 4.1

(a) Complete the following:[31]

<table>
<thead>
<tr>
<th>EQUATION OF THE CURVE</th>
<th>RANGE</th>
<th>AMPLITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \sin x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 2 \sin x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 3 \sin x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = a \sin x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Sketch the curve $y = 5 \sin x$ in the domain $-2\pi \leq x \leq 2\pi$

**Note to Students:**
From this point onwards, **never** sketch a trigonometric function using degrees! It must be in radians.
The Graph of $y = \sin nx$

- The family of curves shown in the diagram above is the result of assigning successive values to $n$.
- The range of each curve is not affected by the value of $n$. As the value of $n$ increases, the period decreases. Hence the value of $n$ determines the period of the function.
- The period for:
  - $y = \sin x$ is $\pi$  
  - $y = \sin 2x$ is $\frac{2\pi}{2} = \pi$ \[32\]  
  - $y = \sin 3x$ is $\frac{2\pi}{3}$ \[33\]  
  - $y = \sin 4x$ is $\frac{2\pi}{4} = \frac{\pi}{2}$ \[34\]

\[
y = \sin nx
\]
Period: $\frac{2\pi}{n}$
Amplitude: 1

Note to Students: Before sketching the graph...
Find how many complete sine (or cosine or tan) cycles your graph needs by finding the period and checking the domain e.g. $0 \leq x \leq 2\pi$
For example:
- $\sin 2x$ has 2 complete cycles in a length of $2\pi$.
- $\sin 3x$ has 3 complete cycles in a length of $2\pi$. 
Concept Check 4.2

(a) Complete the following table. [nx]

<table>
<thead>
<tr>
<th>EQUATION OF THE CURVE</th>
<th>PERIOD</th>
<th>AMPLITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2 \sin 3x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \sin \pi x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 3 \sin \frac{x}{2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Sketch the curve \( y = 3 \sin \frac{2\pi x}{3} \) in the domain \(-3 \leq x \leq 3\)

Did you know:
Your first task is to find the period
(c) The diagram shows the graph of $y = A \sin nx$.

Work out the values of $A$ and $n$. \[36\]
The graph of $y = \cos x$

Note that the graph of $y = \cos x$ is the same as shifting the graph of $y = \sin x$ a distance of $\frac{\pi}{2}$ to the left. This is because $\cos x = \sin \left(\frac{\pi}{2} - x\right)$. For this reason they have the same period and amplitude.

\[ y = \cos x \]

Period: $2\pi$
Amplitude: 1

\[ y = A \cos nx \]

Period: $\frac{2\pi}{n}$
Amplitude: $A$
Concept Check 4.3

(a) Complete the following table. [37]

<table>
<thead>
<tr>
<th>CURVE</th>
<th>PERIOD</th>
<th>AMPLITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 3 \cos 4x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = \frac{\cos \pi x}{4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 2 \cos \frac{3x}{4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) The graph of the curve $y = A \cos n x$ is shown below.

Work out the values of $A$ and $n$. [38]
(c) (i) On the same set of axes draw the graphs of \( y = 3 \sin 2x \) and \( y = 2 \cos x \) in the domain \( -\pi \leq x \leq \pi \).

(ii) Hence write down the number of solutions to the equation \( 3 \sin 2x - 2 \cos x = 0 \) in the domain \( -\pi \leq x \leq \pi \). \([39]\)

(d) Find the maximum and minimum values of \( 4 \cos 2\pi x \) and the values of \( x \) in \( 0 \leq x \leq 2 \) where they occur. \([40]\)

Discussion Question:
If \( x = \sin t \) and \( y = \cos t \) then \( x^2 + y^2 = \sin^2 t + \cos^2 t = 1 \) (unit circle). If we interpret \( t \) as time, how long does it take to go around the circle? \([41]\)
The Graph of $y = A \sin(x - \beta)$ – Horizontal Translations

- Recall from Functions and Relations $y = f(x - \beta)$ is the horizontal translation of $y = f(x)$ by $\beta$ units.
  - If $\beta > 0$, the graph of $y = f(x)$ is shifted to the right by $\beta$ units.
  - If $\beta < 0$, the graph of $y = f(x)$ is shifted to the left by $\beta$ units.
- Similarly $y = A \sin(x - \beta)$ is the graph of $y = A \sin x$ shifted horizontally by $\beta$.
- Note that the period and amplitude of $y = A \sin(x - \beta)$ is the same as $y = A \sin x$.

Example:
Sketch the graph of $y = \cos \left( x - \frac{\pi}{3} \right)$ in the domain $0 \leq x \leq 2\pi$

Step 1: Sketch the graph of $y = \cos x$ in the domain. This will help you sketch your solution

Step 2: Figure out the direction and magnitude the graph has shifted.
$y = \cos \left( x - \frac{\pi}{3} \right)$ shifts to the __________ by __________ radians

Solution:
Concept Check 4.4

(a) Sketch the graph of \( y = 2 \sin \left( x + \frac{\pi}{6} \right) \) in the domain \( 0 \leq x \leq 2\pi \)

(b) Sketch the graph of \( y = 4 \cos(x - \pi) \) in the domain \( 0 \leq x \leq \pi \)
(c) Sketch the graph of $y = 3 \cos \left(2x + \frac{\pi}{2}\right)$ in the domain $0 \leq x \leq \pi$

**Discussion Question:**
Describe the graph of $y = f(2x - a)$. [42]

**Step 1:** Write down the period and amplitude

**Step 2:** Modify the equation to the form $y = a \cos n(x - \beta)$

**Step 3:** Sketch the graph $y = 3 \cos 2x$ and hence the graph of $y = 3 \cos \left(2x + \frac{\pi}{2}\right)$.
Clearly Label both curves.
The graph of \( y = \tan x \)

- When \( x = \frac{\pi}{2} \) and \( \frac{3\pi}{2} \), \( \tan x \) is not defined. This is because \( \tan x = \frac{\sin x}{\cos x} \), and when \( \cos x = 0 \), \( \tan x \) is undefined.
- The graph of \( y = \tan x \) repeats itself after \( \pi \) units.

Discussion Question:
What is the amplitude of \( y = \tan x \)?
Note to Students:
Before you start:
- How many complete tan cycles does your graph need? Find the period and check how many times the period fits in the domain.
  - e.g. if the period was $\pi$ and the domain was $0 \leq x \leq 3\pi$, there will be 3 complete tan cycles in the graph.
- Where are the asymptotes for tan? Draw the asymptotes first and make your graph fit the asymptotes.

Concept Check 4.5
(a) Sketch the graph of $y = \tan 2x$ in the domain $-\pi \leq x \leq \pi$
(b) (i) On the same coordinate axes, sketch the curves $y = \tan \frac{x}{2}$ and $y = 2 \cos 2x$ in the domain $0 \leq x \leq 2\pi$.

(ii) Hence state the number of solutions to the equation $2 \cos 2x = \tan \frac{x}{2}$ [44]
Graphs of Reciprocal Trigonometric Functions

- The graph of \( y = \csc x \) (The reciprocal of \( \sin x \))

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 0 )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \pi )</th>
<th>( \frac{3\pi}{2} )</th>
<th>( 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( \csc x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

- The table shows that asymptotes will occur at \( x = 0, \pi \) and \( 2\pi \).
- The range of the curve is \( y \geq 1 \) or \( y \leq -1 \).

Did you know:
- Check the 3rd letter to find the matching trigonometric functions
  - \( \csc \theta \rightarrow \frac{1}{\sin \theta} \)
  - \( \sec \theta \rightarrow \frac{1}{\cos \theta} \)
  - \( \cot \theta \rightarrow \frac{1}{\tan \theta} \)
- When teachers are looking for harder trigonometric exam questions, they will often use \( \sec x, \csc x \) and \( \cot x \).
Concept Check 4.6

(a) For the curve \( y = \csc x \), write down the period of the function. \[^{45}\]

(b) (i) Complete the following table of values and hence sketch the curve \( y = 2 \csc x \) in the domain \( 0 \leq x \leq 2\pi \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 0 )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \pi )</th>
<th>( \frac{3\pi}{2} )</th>
<th>( 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x )</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \csc x )</td>
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<td></td>
</tr>
<tr>
<td>( 2\csc x )</td>
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</tbody>
</table>

(ii) Write down the range of \( y = 2 \csc x \). \[^{46}\]
(c) (i) Complete the following table of values and hence sketch the graph of $y = \sec x$ in the domain $0 \leq x \leq 2\pi$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>$\frac{\pi}{2}$</th>
<th>$\pi$</th>
<th>$\frac{3\pi}{2}$</th>
<th>$2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sec x$</td>
<td></td>
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</tr>
</tbody>
</table>

(ii) Use the graph to complete the following: $[47$

The range of $y = \sec x$ is

The period of $y = \sec x$ is
(d) (i) Use the following table of values and hence sketch the graph of $y = \cot x$ in the domain $0 \leq x \leq 2\pi$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>$\frac{\pi}{2}$</th>
<th>$\pi$</th>
<th>$\frac{3\pi}{2}$</th>
<th>$2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan x$</td>
<td>0</td>
<td>$\infty$</td>
<td>0</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>$\cot x$</td>
<td></td>
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</tbody>
</table>

(ii) Complete the following: 

The domain of $y = \cot x$ is .................................................................

The range of $y = \cot x$ is .................................................................

The period of $y = \cot x$ is .................................................................
5. PAST H.S.C. QUESTIONS

☐ 2006 H.S.C. Mathematics Q4(a)

In the diagram, $ABCD$ represents a garden. The sector $BCD$ has centre $B$ and $\angle DBC = \frac{5\pi}{6}$.

The points $A$, $B$ and $C$ lie on a straight line and $AB = AD = 3$ metres.

(i) Show that $\angle DAB = \frac{2\pi}{3}$

(ii) Find the length of $BD$ [49]
(iii) Find the area of the garden $ABCD$ \[50\]

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct answer</td>
<td>2</td>
</tr>
<tr>
<td>Finds the area of sector $BCD$ or equivalent merit</td>
<td>1</td>
</tr>
</tbody>
</table>

NOTES FROM THE MARKING CENTRE

(i) In better responses, candidates gave reasons for the three geometrical steps and used the diagram to illustrate their thought process by marking in the angles. Most candidates worked with radians, though some chose to convert to degrees first before proceeding with their solutions. Candidates using a mixture of radians and degrees in the same equation did not gain the mark.

(ii) Candidates who used the correct cosine rule and substituted correctly usually scored full marks. The sine rule was also used by candidates with the same degree of success. Dividing the isosceles triangle into right-angle triangles for calculation of BD was another method used. Loss of marks usually resulted from incorrect use of calculators, a wrong angle, or incorrect cosine rule or sine rule.

(iii) Candidates who knew their formulae were generally successful in obtaining the correct answer. The most common errors were not knowing the sector area formula, and leaving out the $\frac{1}{2}$ in the sector area formula or the formula for the area of the triangle. Another error was the use of 150 instead of $\frac{5\pi}{6}$ in the sector area formula. Some candidates were unable to simplify or work with surds and fractions, resulting in calculation errors in their responses.
2010 H.S.C. Mathematics Q8(c)

The graph shown is $y = A \sin bx$.

(i) Write down the value of $A$. [51]

(ii) Find the value of $b$. [52]

(iii) On the same set of axes, draw the graph $y = 3 \sin x + 1$, for $0 \leq x \leq \pi$.

Criteria | Marks
--- | ---
Correct graph | 2
Sine curve showing correctly any two fo period, amplitude, mean value | 1