YEAR 11
MATHS EXT 1

LESSON 7: QUADRATIC POLYNOMIAL 1
Notice to Students

- Attempt ALL questions.
- If you are having difficulty with your Work Booklet, please seek assistance through WORKSHOPS.
- Your teacher will check that you have completed this work booklet in the assigned time.
- Allocated time for this Work Booklet is 90 minutes.
Multiple Choice Questions

1. What values of $m$ does the quadratic equation $x^2 - (3 + m)x + 4m = 0$ have no real roots?[1]
   - (A) $m < -3$ or $m > 0$
   - (B) $-3 < m < 0$
   - (C) $m < 1$ or $m > 9$
   - (D) $1 < m < 9$

2. What are the conditions for the expression $ax^2 + bx + c$ to be positive definite? [2]
   - (A) $a > 0$ and $\Delta > 0$
   - (B) $c > 0$ and $\Delta > 0$
   - (C) $a > 0$ and $\Delta < 0$
   - (D) $c > 0$ and $\Delta < 0$

3. If $p$ and $q$ are the roots of $15x^2 + 75x - 3 = 0$ then $p + q = $ [3]
   - (A) 75
   - (B) 5
   - (C) $-\frac{1}{5}$
   - (D) $-5$

4. The quadratic function $Q(x) = 5x^2 - 4x + 3$, has roots for $Q(x) = 0$ of $\alpha$ and $\beta$. [4]
   Hence, $\alpha^2 + \beta^2 =$
   - (A) $\frac{46}{25}$
   - (B) $\frac{29}{25}$
   - (C) $-\frac{11}{25}$
   - (D) $-\frac{14}{25}$
Question 1

Without solving the following equations state whether the roots are:

(i) real or unreal.
(ii) equal or unequal, rational or irrational when they are real.

(a) $4x^2 - 4x + 1 = 0$ [5]

(b) $3x^2 - 5x + 5 = 0$ [6]

(c) $x^2 - 3 = 0$ [7]

(d) $2x^2 - 5x - 4 = 0$ [8]

(e) $8 - 4x - 5x^2 = 0$ [9]
Question 2

(a) The quadratic equation \((k - 4)x^2 - 2x - 1 = 0\) has equal roots.

(i) Find the values of \(k\).  \([10]\]

(ii) Determine the roots for this value of \(k\).  \([11]\]

(b) The equation \(x^2 + kx + 16 = 0\) has real and distinct roots. Find the value of \(k\).  \([12]\]

(c) Without solving the equation \(3x^2 - 7x + 5 = 0\), describe the characteristics of the roots of this equation.  \([13]\]

(d) The expression \(x^2 + (2 - k)x + k(2 - k)\) is positive for all values of \(x\). Find the value of \(k\).  \([14]\]

Note to Students:
- \(a > 0\)
- \(\Delta > 0\)
(e) Show that \( x^2 + 2x + 5 > 0 \) for all values of \( x \).

Question 3

The line \( y = mx - 4 \) and the parabola \( y = x^2 + 5 \) are drawn on the same number plane. Find the values of \( m \) if the line and the parabola:

- (a) intersect at two points. \([15]\)
- (b) touch at one point. \([16]\)
- (c) do not intersect. \([17]\)

**Note to Students:**

\[
\begin{align*}
x^2 + 5 &= mx - 4 \\
x^2 - mx + 9 &= 0 \\
a &= 1, b = -m, c = 9
\end{align*}
\]
Question 4

(a) Show that the line $kx - y + 2 - k = 0$ is not a tangent to the circle $x^2 + y^2 = 9$ for any value of $k$.

(b) The quadratic expression $(2 - m)x^2 + 2(1 - m)x + m - 1$ is positive definite. Find the possible values of $m$. [18]

Note to Students:
- Positive definite $\rightarrow$ $y$ values are definitely positive
- Negative definite $\rightarrow$ $y$ values are definitely negative
Question 5

(a) If \( x = 1 \) is a root of the equation \( kx^2 - x - 2 = 0 \), find the other root. \([19]\)

(b) Consider the equation \( x^2 + kx - 5k - 1 = 0 \). Find the value of \( k \) if the roots are:

(i) reciprocals of one another. \([20]\)

(ii) equal but opposite in signs. \([21]\)

(c) The equation \( 3x^2 + 5x + 1 = 0 \) has one root \( x \approx -0.23 \) (correct to 2 decimal place). Without solving the equation, find the other root, correct to 2 decimal places. \([22]\)
(d) The equation \((2k + 3)x^2 + (k - 2)x + (3k - 1) = 0\) has roots equal in magnitude but opposite in signs. Find the value of \(k\). \([23]\)

**Note to Students:**
Let the roots be \(\alpha, -\alpha\).

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**Question 6**

\(\alpha\) and \(\beta\) are the roots of the equation \(3x^2 - 4x - 1 = 0\). Find the values of:

(a) \(\alpha^2 + \beta^2\) \([24]\)

(b) \(\frac{1}{\alpha} + \frac{1}{\beta}\) \([25]\)

(c) \(\frac{1}{\alpha + 2} + \frac{1}{\beta + 2}\) \([26]\)
Question 7

Find the relationship between $a$, $b$ and $c$ in the equation $ax^2 + bx + c = 0$ when the:

(a) larger root exceeds the smaller root by 1.  \[27\]

(b) roots are in the ratio 2:3.  \[28\]

Question 8

Three times the sum of the roots of the equation $x^2 - (a^2 + b^2)x + ab = 0$ is equal to ten times their product. Find a possible relationship between $a$ and $b$.  \[29\]
Question 9
Find the values of \( k \) in the equation if \( x^2 - (k + 2)x + 4k = 0 \) has:

(a) one of the two roots equal to \(-5\). \(^{[30]}\)

(b) the sum of the two roots equal to 7. \(^{[31]}\)

(c) one root 2 more than the other. \(^{[32]}\)

(d) one root the negative reciprocal of the other. \(^{[33]}\)
Question 10

Form possible quadratic equations with the given roots:

(a) \(-5 \text{ and } 7\)  

(b) \(3 \pm \sqrt{7}\)

(c) \(-a \pm \sqrt{a^2 - b}\)

(d) \(\frac{m}{n} \text{ and } \frac{-n}{m}\)
Question 11

(a) \( \alpha \) and \( \beta \) are the roots of the equation \( 3x^2 + 2x + (2 - p) = 0 \). Find value of \( p \) when \( \alpha^2 + \beta^2 = \frac{4}{9} \). \([38]\]

(b) The chord \( 3x - 2y - 8 = 0 \) intersects the parabola \( y = \frac{x^2}{8} \) at the points P and Q. Find the midpoint of PQ without finding the coordinates of P or Q. \([39]\]

Note to Students:
This is an important skill!
Remember we can find \( \alpha + \beta \) without knowing \( \alpha \) or \( \beta \). Therefore we can find \( \frac{\alpha + \beta}{2} \) as well.