YEAR 11
MATHS EXT 1
ACCELERATION

LESSON 4: PARAMETRIC REPRESENTATION 1
(Preliminary Mathematics Ext 1 Topic)
Notice to Students

- Attempt ALL questions.
- If you are having difficulty with your Work Booklet, please seek assistance through WORKSHOPS.
- Your teacher will check that you have completed this work booklet in the assigned time.
- Allocated time for this Work Booklet is 90 minutes.
Question 1

Find the Cartesian equations for the following curves with parametric equations:

(a) \( x = 8t - 5; \ y = t^2 + 2t - 3 \) \([1]\) 

(b) \( x = 2 \sec \theta; \ y = 3 \tan \theta \) \([2]\) 

(c) \( x = 2t; \ y = t^2 + 2 \) \([3]\) 

(d) \( x = \sin \theta; \ y = \cos^2 \theta - \sin^2 \theta \) \([4]\)
Question 2

(a) (i) Show that \((6t - 5, 3t^2 - 6)\) lies on the parabola \((x + 5)^2 = 12(y + 6)\)

(ii) Find the coordinates of \(P\) where \(t = -2\) \([5]\)

(b) A parabola has parametric equations \(x = 4t\) and \(y = 2t^2\). Find the:
   (i) coordinates of its focus \([6]\)
   (ii) equation of its directrix \([7]\)

(c) (i) Find the coordinates of the point \(P\) on the parabola \(x = t^2, y = -2t\) where \(t = 2\). \([8]\)
   (ii) Find the equation of the line \(PS\) where \(S\) is the focus of the parabola. \([9]\)
Question 3

\( P(2ap, ap^2) \) and \( Q(2aq, aq^2) \) are two points on the parabola \( x^2 = 4ay \). Find the:

(a) equation of the chord \( PQ \) \[10\]

(b) coordinates of the point of intersection of the chord \( PQ \) and the directrix of the parabola \[11\]

(c) coordinates of \( Q \) in terms of \( p \) if \( PQ \) is a focal chord. \[12\]
Question 4

\(P(8p, 4p^2)\) and \(Q(8q, 4q^2)\) are two points on the parabola \(x^2 = 16y\).

(i) Find the equation of the chord \(PQ\). \([13]\]

(ii) Find the coordinates of the point where the chord meets the directrix. \([14]\]

(iii) Find the relationship between \(p\) and \(q\) if \(PQ\) is a focal chord. \([15]\]
Question 5

\( (2ap, ap^2) \) is a point on the parabola \( x^2 = 4ay \). The tangent at \( P \) meets the directrix of the parabola at \( T \). \( S \) is the focus of the parabola.

(i) Find the equation of the tangent at the point \( P \). [16]

(ii) Find the coordinates of \( T \). [17]

(iii) Show that \( ST \) is perpendicular to \( SP \).
Question 6

$p(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.

(i) Find the equation of the tangent to the parabola at the point $P$. [18]

(ii) The tangent at $P$ meets the $x$-axis at $T$ and the $y$-axis at $U$. Find the coordinates of the points $T$ and $U$. [19]

(iii) $S$ is the focus of the parabola. Show that $ST$ is perpendicular to $PT$.

(iv) Show that $T$ bisects $PU$. 
Question 7

\[ x^2 = 4ay \]

\( \text{P(2ap, ap^2)} \) and \( \text{Q(2aq, aq^2)} \) are two points on the parabola \( x^2 = 4ay \). \( T \) is the point of intersection of the tangents drawn at \( P \) and \( Q \). \( N \) is the point of intersection of the normals drawn at \( P \) and \( Q \).

(i) Find the equation of the tangent and normal at \( P \). [20]

(ii) Hence find the coordinates of \( T \) and \( N \). [21]
(iii) Show that \(TN\) is parallel to the axis of the parabola if \(PQ\) is a focal chord.

(iv) If \(pq = 2\), show that \(N\) lies on the parabola.
Question 8

Two points $P(2Ap, Ap^2)$ and $Q(2Aq, Aq^2)$ lie on the parabola $x^2 = 4Ay$, where $A > 0$. The chord $PQ$ passes through the focus.

(a) Show that $pq = -1$

(b) Show that the point of intersection $T$ of the tangents to the parabola at $P$ and $Q$ lies on the line $y = -A$

(c) Show that the chord $PQ$ has length $A(p + \frac{1}{p})^2$
Question 9

In the diagram given below, $Q(4q, 2q^2)$ is a point on the parabola $x^2 = 8y$. $BQ$ is drawn parallel to the $y$-axis and $CT$ is a tangent to the parabola at $Q$. $S$ is the focus of the parabola and $T$ is the $y$-intercept of the tangent at $Q$.

(a) Find the equation of the tangent at $Q$. [22]

(b) Show that $T$ has coordinates $(0, -2q^2)$.

(c) Show that $SQ = ST$

(d) Hence or otherwise prove that $\angle CQB = \angle SQT$. 